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The Consequences of Tax Reforms

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Abstract

In this paper, I examine the effects of tax reforms for an economy in a welfare state. I do this by a macroeconomic model, which includes not only households, firms and a government but also a monopoly labour union. The households are of two types, workers and entrepreneurs. The workers are employed by the firms, which are owned by the entrepreneurs. This paper shows that decreasing labour taxation and increasing consumption taxation would have a number of positive effects. These include increased aggregate utility as well as increased consumption of the workers combined with increased profits of the entrepreneurs. Also the employment rate would improve. Decreasing profit taxation and increasing consumption taxation would also have positive effects, but only for the entrepreneurs.

JEL Classification: H20, H24, H25, H31, H32

Keywords: taxation, tax reform, utility

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1 List of Variables and Parameters

A production level parameter

C workers' consumption

D government debt

E employment rate

F level of production

G public expenditures

I entrepreneurs' investment

K entrepreneurs' capital stock

L entrepreneurs' labour demand

N workers' labour supply

P price level

T_{LW} worker's labour tax

T_{π} profit tax rate

T_C consumption tax rate

U_W workers' utility

U_E entrepreneurs' utility

W workers' wealth/savings

w gross wage rate

Y entrepreneurs' gross income

α ratio between unemployment benefits and wage rate

δ depreciation rate

η income share of capital

π entrepreneurs' profits

ρ rate of time preference

σ factor elasticity

ψ worker's dis-utility of working

2 Introduction

Finland, being one of the Nordic welfare states, is generally considered as an example of a society with high taxation. However, there are signs that the taxation has reached such a high level that many politicians prefer not to cross. In spite of that public expenditures tend still to rise and there is a discussion about how to deal with the situation taking into consideration a socially acceptable solution. One result seems to be that important sectors of society, e.g. social security and education, suffer. Instead politicians should be open-minded regarding other solutions. Those could involve changing the balance between different forms of taxation. However, it could also be interesting to investigate the results of smaller public expenses combined with a lower general level of taxation. My research topic is *how reforms of the tax code, including possible changes in public expenditures, impact the welfare of individuals in society.* The welfare is measured by utility level and income. I investigate the theoretical impacts in a steady state environment.

In the USA, one of the big tax reforms was "The Tax Reform Act of 1986", approved during the presidency of Ronald Reagan. The reform simplified the system and concerned persons as well as companies.[1] However, during the years a number of amendments changed the original intentions. In 1997 during the Bill Clinton administration, the "Taxpayer Relief Act" was passed and that has generally been considered to complicate the system [2]. After the election of President Donald Trump in 2016, the Republican Party has controlled the presidency as well as the Congress. The party used in December 2017 its power to change the taxation system by approving the "Tax Cuts and Jobs Act" decreasing temporarily personal income tax rates and permanently the corporate tax rate [3].

[1] Chamberlain, Andrew. Twenty Years Later: The Tax Reform Act of 1986. Tax Foundation. 2006.

[2] Altig, David, Auerbach, Alan J., Kotlikoff, Laurence J., Smetters, Kent A. and Walliser, Jan. Simulating Fundamental Tax Reform in the United States. *The American Economic Review* 91, no. 3 (2001): 574-595.

[3] Tax Foundation. Preliminary Details and Analysis of the Tax Cuts and Jobs Act. 2017.

Also in Europe there have during the last decades been substantial changes in the tax system. The Baltic countries were in 1994 and 1995 pioneers regarding introducing different forms of so called flat tax systems [4]. The background is described for instance by Hall and Rabushka [5]. The system can simply be a flat tax for all kinds of income, personal as well as corporate, or refined with various exemptions including deductions and income levels. Regarding Finland one of the most interesting changes regarding the taxation was the one lowering the corporate tax in 2014 from 24.5 to 20 percentage [6].

The literature dealing with taxation is very extensive. It includes articles and books which concentrate on a specific form of taxation as well as those which present a broader approach. The literature can also be categorized based on whether the labour market is unionized or not. Also the degree of the unionization varies. Below some of the relevant contributions are briefly discussed. The material is ordered based on the year of publication and not on the relevance for my research.

Altig et al. (2001) propose a dynamic macroeconomic model to simulate the impact and efficiency of various tax regimes. The model of Altig et al. considers households, firms and the government in an intragenerational setup. The consequences of a proportional income tax, a proportional consumption tax, flat taxes etc. are examined. In the setup there are "winners" and "losers". The conclusion is that in some cases retired people loose when the situation of future generations is improved. In other cases middle- and upper-income citizens will be better off and current and future poor people will suffer. In line with the article, the big question is "are the gains to the winners worth the costs for the losers".[7]

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[4] European Central Bank. Economic and monetary developments. Monthly Bulletin. 2007

[5] Hall, R. E. and Rabushka, A. *Low Tax, Simple Tax, Flat Tax.* New York: McGraw Hill, 1983.

[6] Veronmaksajat. Yhteisöverotus. `https://www.veronmaksajat.fi/luvut/Tilastot/Tuloverot/Yhteisoverotus/`. 2017. Accessed: February 2018.

[7] Altig et al., ibid.

Aronsson et al. (2002) propose a general equilibrium model based on the idea that the labour unions determine about the wages. The agents in the model are the households/labour unions, the firms and the government. The taxes are forwarded to the households as a lump-sum transfer. It is assumed that the size of the population does not change. The paper researches the effects of increasing the progressivity of taxation on the main variables in the economy. Among the conclusions of the paper are that real wages are increased and other variables such as working time, employment, output and consumption are decreased.[8]

Aronsson and Sjögren (2003) use a model for a small open economy. The world market determines the prices of products. The players are the consumers of two productivity types, deciding about the labour supply, the firms deciding about the labour demand, the labour unions deciding about the wages and the government choosing the tax rates and the public expenditures. The labour supply exceeds the demand because of the wages set by the labour unions. One result of the investigation is that if the unemployment benefits are low, then the level of provided public goods are increased and the commodity taxes are decreased. Another conclusion is that if the wage ratio between the two consumer types change, the labour input of the consumer types react similarly.[9]

Aronsson and Sjögren (2004) base their research on a general equilibrium model of a unionized economy. The agents are three consumer types i.e. the owners of the firms and employed as well as unemployed workers, identical firms, labour unions and a utilitarian government. The production only depends on labour and the labour unions either decide only about the wage rates or also about the number of working hours per worker. The paper shows that in case the labour unions fix only the wages and in case the income tax is unrestricted, the free market solution, including zero marginal

[8] Aronsson, Thomas, Löfgren, Karl-Gustaf and Sjögren, Tomas. Wage setting and tax progressivity in dynamic general equilibrium. *Oxford Economic Papers* 54 (2002): 490-504.

[9] Aronsson, Thomas and Sjögren, Tomas. Income Taxation, Commodity Taxation and Provision of Public Goods under Labor Market Distortions. *Finanzarchiv* 59(3) (2003): 347-370.

income tax, can be implemented. On the other hand if the income taxation is restricted, there will be unemployment and a progressive labour income tax. In the case of the labour unions choosing also the working hours there is direct influence on the progressivity.[10]

Paulus and Peichl (2009) use the so called Euromod model, which is a tax-benefit framework suitable for making comparative analysis for the EU economies, to simulate the effects of flat tax reforms in Western Europe. The Euromod model used by Paulus and Peichl makes it possible consistently to compare the countries concerned. The authors come to the conclusion that in certain situations a flat tax could increase income equality and encourages people to work more. The real consequences however depend on the design of the tax reform and what country is concerned. The simulated reforms resemble the flat tax schemes in Eastern Europe.[11]

Díaz-Giménez and Pijoan-Mas (2011) describe a modified neoclassical growth model where the households are heterogeneous and cannot insure themselves against idiosyncratic risks. The authors study the effects of various flat-tax reforms on the distribution of income and the welfare of individuals in a model economy. The model of Díaz-Giménez and Pijoan-Mas use stochastic aging and retirements to shape features connected to the life-cycle of individuals. The government in the model taxes capital income, labour income, consumption and estates. The taxes are used for the benefits given to retired households and for government consumption. According to the findings a move towards a progressive consumption-based flat tax actually raises the government income as the economy is stimulated. At the same time it supports the weakest people in the society.[12]

Aronsson and Wikström (2011) introduce a research based on Aronsson and Sjögren (2004) with the same agents of the model. In the paper of 2011

[10] Aronsson, Thomas and Sjögren, Tomas. Is the Optimal Labor Income Tax Progressive in a Unionized Economy?. *Scandinavian Journal of Economics* 106(4) (2004):661-675.

[11] Paulus, Alari and Peichl, Andreas. Effects of flat tax reforms in Western Europe. *Journal of Policy Modeling* 31 (2009): 620-636.

[12] Díaz-Giménez, Javier and Pijoan-Mas, Josep. Flat Tax Reforms: Investment Expensing and Progressivity. *CEMFI Working Paper* nr. 1101. (2011).

the research is concentrating on how the progressivity of the labour income taxation is impacted by whether the waging setting of the labour unions is either centralized or decentralized. The utilitarian government now applies two types of taxes, an unrestricted tax of profits and a progressive tax on working income. The paper shows that with a decentralized wage setting, the free market solution is attainable contrary to the centralization which leads unemployment and a progressive taxation.[13]

Piketty et al. (2014) introduce a model for an optimal and a maximum tax rate on working income. The authors develop optimality equations for the maximal tax rates. The model of Piketty et al. considers three possible responses to the tax policy. These responses are the supply-side, the tax-avoidance and the compensation-bargaining response. The authors develop optimality equations for the maximal tax rates. These equations depend on the responses mentioned above. The conclusions are that the supply-side response leads in theory to decreased tax rates being optimal for top earners as well as the others. The authors however, present some reasons why this result in practice is problematic. In the case of tax-avoidance responses the result is that high income individuals try to use the system in order to avoid taxes. The authors suggest that these loopholes should be closed and that top taxes can then be increased. Finally the compensation-bargaining response encourages top earners to bargain for even higher income if their taxes are lowered.[14]

Hummel and Jacobs (2016) analyze an economy that takes in consideration workers, trade unions, owners of firms and the government. In the model the labour market is unionized, workers face heterogeneous labour participation costs and the wage rate depends on the type of work. Workers decide if they want to take part in the labour market. Workers in different sectors belong to different labour unions. The owners of the firms have a capital stock and

[13]Aronsson, Thomas and Wikström, Magnus. Optimal Tax Progression: Does it Matter if Wage Bargaining is Centralized or Decentralized?. Department of Economics, Umeå University. 2011.

[14]Piketty, Thomas, Saez, Emmanuel and Stantcheva, Stefanie. Optimal Taxation of Top Labor Incomes: A Tale of Three Elasticities. *American Economic Journal* 6, no. 1 (2014): 230-271.

they hire different types of labour. The trade unions and the owners of the firms negotiate about the wages. The conclusions are for instance that because labour unions often raise the wages above the market level the result is involuntary unemployment and that the unionization of the labour market decreases the optimal income taxation.[15]

In this paper I create a macroeconomic model, which can be used for calculating and estimating the effects of possible tax reforms for the individuals. The model consists of heterogenous households, firms, a monopoly labour union and a government. The households are of two types, those who own the firms and those who work in the firms. The model is not based on any specific existing model, although, I include features from certain macroeconomic models introduced in existing papers. In practice this means that I introduce a unique model to be used in the discussion about taxation in society.

I start by describing the model in chapter 3. I present a general as well as a specified description of the players involved. In chapter 4 I describe the steady state of the model and in chapter 5 I make the welfare analysis concerning the households. In the analysis I deal with a number of different tax reform proposals and present the impacts of those. Finally in chapter 6 I draw conclusions of the results of my research.

3 The Model

The model consists of the government, the labour union, the firms and the households and takes the form of a Stackelberg game. First the government acts, followed by the labour union. Then the firms and the households act simultaneously.

The model is solved backwards starting from the households and the firms, followed by the labour union and ending with the government. In each step

[15] Hummel, Albert Jan and Jacobs, Bas. Optimal Income Taxation in Unionized Labor Markets. Erasmus University Rotterdam and Tinbergen Institute. (2016).

when making the calculations, the optimality conditions of the previous steps are treated as existing constraints.

3.1 General description of the agents

Two types of households are assumed to exist - those who work in the firms ("workers") and those who are the owners of the firms ("capitalists"). The capitalists and firms are merged into one agent ("entrepreneur") and are inseparable from each other.

There is a finite number N of identical workers. They are either employed or unemployed. An employed worker supplies one unit of labour each period. A worker earns either a working income w_t, paid by a firm or receives an unemployment benefit $B_t(w_t) = \alpha w_t$, paid by the government, where $\alpha \in (0,1)$. The representative household decides how much to consume, C_t. The rest of the wealth, i.e. the savings, W_{t+1}, is invested in one-period bonds issued by the government. In the next period the representative household receives from the government the value of the bonds including a periodic interest rate r_{t+1}, i.e. totally $(1 + r_{t+1})W_{t+1}$, i.e. the real interest rate in period t being r_t, where $r_t \in (-\infty, \infty)$. The consumption is taxed at the rate $T_{C,t}$ and the labour income is taxed at the rate $T_{LW,t}$, where $T_{C,t}, T_{LW,t} \in (-\infty, 1)$. $T_{LW,t}$ includes the ordinary labour income tax as well as the social security tax, paid by the worker. This labour tax is assumed to be deducted from the income by the employing firm, which transfers the deducted sum to the government.

The entrepreneurs are assumed to be identical and to own the capital stock. Entrepreneurs produce a homogeneous final good using the production technology $F(K, L)$, which exhibits constant returns-to-scale with regard to capital K and labour L. In every period t, the entrepreneurs make investment decisions by choosing the next period's capital stock K_{t+1}. Further, given the gross wage w_t of the worker in the period concerned, and the current period's capital stock K_t, entrepreneurs decide how much labour L_t to employ in period t. The price for the final good is P_t, where $P_t > 0$. The investments are assumed to consist of the final good. The total profits of the entrepreneurs are partly used for the investments and partly for consumption. As the discrimination between the investments and the consumption of entrepreneurs is incentive incompatible for the government, both are taxed as consumption, i.e. at the rate $T_{C,t}$. The investments are denoted by I_t. For simplicity, the labour costs as well as the costs of investments are deducted from the gross income of the entrepreneur. The remaining part of the gross income is taxed at the rate $T_{\pi,t}$, where $T_{\pi,t} \in (-\infty, 1)$. The entrepreneurs' net income is then denoted by π_t and is for simplicity in this paper called "profit".

The workers are represented by a monopoly labour union. The labour union chooses the wage rate that maximizes the representative worker's lifetime utility, taking into consideration the worker's budget constraint and the entrepreneur's capital stock and labour demand decisions.

The government collects taxes and receives revenue by issuing bonds, the amount being D_{t+1} in period t. It uses the income to finance the public expenditures G_t, the unemployment benefits $B_t(w_t)$ and the repayment of old bond debts including the corresponding interest, i.e. $(1 + r_t)D_t$. The tax rates are exogenous and the public expenditures are determined through the government's budget constraint. The government's budget is assumed to be balanced.

3.2 Specified description of the agents

In order to solve the model, a more detailed description of the players is needed.

3.2.1 Workers

The probability of a worker being employed is $\frac{L_t}{N}$, where L_t is the total labour demand and N is the total labour supply. The worker's marginal disutility of working is $\psi > 0$. The representative worker derives utility from his consumption, C_t, and leisure time, $N - L_t$, i.e. the time he is not working. The worker's utility function is logarithmic and therefore the periodic utility function of the representative worker in period t is

$$U_W(C_t, L_t) = \log(C_t) + \psi \log(N - L_t). \tag{1}$$

The lifetime utility of the representative worker is

$$\sum_{t=0}^{\infty} \beta^t \big(\log(C_t) + \psi \log(N - L_t)\big), \tag{2}$$

where $\beta = \frac{1}{1+\rho}$ is the discount factor, $\rho > 0$ being the rate of time preference.

Based on the discussion in chapter 3.1, the budget constraint is

$$\begin{aligned}&(1 + r_t)W_t + \frac{L_t}{N} w_t(1 - T_{LW,t})N + (1 - \frac{L_t}{N})\alpha w_t(1 - T_{LW,t})N \\ &= (1 + T_{C,t})P_t C_t + W_{t+1},\end{aligned} \tag{3}$$

or equivalently

$$\begin{aligned}C_t = &\frac{(1 + r_t)W_t}{P_t(1 + T_{C,t})} + \frac{L_t w_t(1 - T_{LW,t})}{P_t(1 + T_{C,t})} \\ &+ \frac{(N - L_t)\alpha w_t(1 - T_{LW,t})}{P_t(1 + T_{C,t})} - \frac{W_{t+1}}{P_t(1 + T_{C,t})}.\end{aligned} \tag{4}$$

The worker's utility maximization with respect to consumption is based on dynamic programming. This is done in Appendix C.1.

Based on the calculations, the consumption path of the representative household is determined by the Euler equation

$$\frac{C_{t+1}}{C_t} = \beta \frac{P_t(1 + T_{C,t})}{P_{t+1}(1 + T_{C,t+1})}(1 + r_{t+1}). \tag{5}$$

3.2.2 Entrepreneurs

The representative entrepreneur derives utility from his profit, π_t. The entrepreneur's utility function is logarithmic and therefore the periodic utility function of the representative entrepreneur in period t is

$$U_E(\pi_t) = \log(\pi_t). \tag{6}$$

The lifetime utility of the representative entrepreneur is

$$\sum_{t=0}^{\infty}(\frac{1}{1+r_t})^t \log(\pi_t). \tag{7}$$

The capital accumulation of the entrepreneurs can be described by the following equation:

$$K_{t+1} - K_t = I_t - \delta K_t, \tag{8}$$

where δ is the depreciation rate of capital, assuming that $\delta \in [0,1]$. Hence the invested amount equals the difference between next period's amount of capital and what is left of the current period's capital after depreciation, i.e.

$$I_t = K_{t+1} - (1-\delta)K_t. \tag{9}$$

The gross income of the representative entrepreneur, Y_t, is defined as

$$Y_t = P_t F(K_t, L_t), \tag{10}$$

which is the measurement for the Gross Domestic Product.

The total periodic profits of the entrepreneurs can therefore be expressed as

$$\pi_t = (1 - T_{\pi,t})\big[P_t F(K_t, L_t) - w_t L_t - (1 + T_{C,t})P_t I_t\big], \tag{11}$$

which the representative entrepreneur considers to be his budget constraint.

In this model the production function $F(K_t, L_t)$ is defined as

$$F(K_t, L_t) = A[\eta K_t^{\frac{\sigma-1}{\sigma}} + (1-\eta)L_t^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}, \tag{12}$$

where $\eta, \sigma \in (0,1)$. The parameter η is distribution parameters, determining the distribution of income between the factors of production. The parameter σ measures the elasticity of factor substitution.

The representative entrepreneur in period t chooses, as mentioned earlier, next period's capital stock K_{t+1} and the labour demand in the current period, L_t, in order to solve its maximization problem. The entrepreneur's maximization is based on dynamic programming, which is done in Appendix C.2.

Based on the calculations, the representative entrepreneur's optimal decision concerning the capital stock in the next period is determined by the equation

$$\begin{aligned} &\frac{(1-T_{\pi,t+1})P_{t+1}}{(1+r_{t+1})\pi_{t+1}}\Big[F_K(K_{t+1},L_{t+1})+(1+T_{C,t+1})(1-\delta)\Big] \\ &=\frac{(1-T_{\pi,t})(1+T_{C,t})P_t}{\pi_t}. \end{aligned} \tag{13}$$

where the expression for $F_K(K_t, L_t)$ is given by equation (41) in the appendix.

Based on the calculations, the representative entrepreneur's optimal decision concerning the labour demand in the current period is determined by the equation

$$P_t F_L(K_t, L_t) = w_t, \tag{14}$$

where the expression for $F_L(K_t, L_t)$ is given by equation (48) in the appendix.

3.2.3 Labour union

The labour union maximizes the lifetime utility of the representative worker, which is based on the workers' periodic utility function given by equation (1), with respect to the wage rate w_t, subject to the workers' budget constraint (4), the capital stock decision and the labour demand of the representative entrepreneur. The labour union's maximization with respect to w_t is solved in Appendix C.3.

Based on the calculations, the labour union's optimal decision concerning the wage rate in the current period is determined by the equation

$$\begin{aligned} &\big[L_t+(N-L_t)\alpha\big](1-T_{LW,t})(N-L_t)P_tF_{LL}(K_t,L_t) \\ &+(1-\alpha)(N-L_t)w_t(1-T_{LW,t})=\psi(1+T_{C,t})P_tC_t. \end{aligned} \tag{15}$$

where the expression for $F_{LL}(K_t, L_t)$ is given by equation (54) in the appendix.

3.2.4 Government

The budget constraint for the government is

$$\begin{aligned} & T_{\pi,t}\big\{P_t F(K_t, L_t) - w_t L_t - (1+T_{C,t})P_t\big[K_{t+1} - (1-\delta)K_t\big]\big\} \\ & + T_{C,t}P_t\big(C_t + K_{t+1} - (1-\delta)K_t\big) + T_{LW,t}w_t L_t \\ & + D_{t+1} = (1+r_t)D_t + P_t G_t + (N - L_t)\alpha w_t(1 - T_{LW,t}), \end{aligned} \tag{16}$$

where the expression for $F(K_t, L_t)$ is given by equation (12).

The taxes, paid by the workers, are

$$T_{C,t}P_t C_t \quad \text{and} \quad T_{LW,t}w_t L_t$$

The taxes, paid by the entrepreneurs, are

$$\begin{aligned} & T_{\pi,t}\big\{P_t F(K_t, L_t) - w_t L_t - (1+T_{C,t})P_t\big[K_{t+1} - (1-\delta)K_t\big]\big\} \\ & \text{and} \quad T_{C,t}P_t\big(K_{t+1} - (1-\delta)K_t\big) \end{aligned}$$

The expenditures for the government are the public expenses $P_t G_t$ and the unemployment benefits $(N - L_t)\alpha w_t(1 - T_{LW,t})$, paid to the workers.

3.2.5 The Bonds Market

It is assumed that the entrepreneurs do not issue bonds. Therefore the only bonds issuer is the government, the amount being D_{t+1} in period t. The only agent buying bonds is the workers, the amount being W_{t+1} in period t. The bonds market in period t is characterized by the following equation:

$$D_{t+1} = W_{t+1}. \tag{17}$$

3.2.6 The Goods Market

The model presented above fulfills Walras' law. This is shown in appendix B. The equilibrium of the goods market is

$$P_t F(K_t, L_t) = P_t C_t + P_t G_t + \pi_t + P_t\big(K_{t+1} - (1-\delta)K_t\big). \qquad (18)$$

4 Description of the steady state

The model is examined in the steady state of the system (3), (5), (11), (13), (14), (15) and (18). These steady state equations are developed in Appendix $D.1$. Here the government's budget equilibrium (16) is left out, referring to Walras' law. The endogenous variables are the capital stock and labour demand of the entrepreneurs, the wage rate chosen by the monopoly labour union, the consumption of the workers, the profit of the entrepreneurs and finally the savings of the workers, i.e. $\{K^*, L^*, w^*, C^*, \pi^*, W^*\}$. The exogenous variables are the tax rates on labour, profit and consumption as well as the ratio between government expenditures and the capital stock, i.e. $\{T^*_{LW}, T^*_{\pi}, T^*_C, G^*\}$.

5 Analysis

When considering different tax reform proposals, it is essential to examine the possible effects. It is certainly important to try to foresee how the utility of an individual is going to react to tax changes. This paper additionally examines the effects on the workers' consumption, the entrepreneurs' profits, the employment rate and the capital stock.

Calculations made in Appendices $D.2$ - $D.3$ are used in this chapter.

The utility of the representative worker increases with the worker's consumption and decreases with the labour demand of the entrepreneur as leisure time gives the worker positive utility. Thus the following holds:

$$U_W^*(C^*, L^*), \quad \text{where} \quad \frac{\partial U_W^*}{\partial C^*} > 0 \quad \text{and} \quad \frac{\partial U_W^*}{\partial L^*} < 0. \tag{19}$$

The utility of the representative entrepreneur increases with the entrepreneur's profit, i.e.

$$U_E^*(\pi^*), \quad \text{where} \quad \frac{\partial U_E^*}{\partial \pi^*} > 0. \tag{20}$$

5.1 Individual tax effects

The capital stock decision of the representative entrepreneur is described by the following function:

$$\begin{aligned} &K^*(T_{LW}^*, T_\pi^*, T_C^*, G^*), \quad \text{where} \quad \frac{\partial K^*}{\partial T_{LW}^*} < 0, \\ &\frac{\partial K^*}{\partial T_\pi^*} < 0, \quad \frac{\partial K^*}{\partial T_C^*} < 0, \quad \frac{\partial K^*}{\partial G^*} > 0. \end{aligned} \tag{21}$$

A higher tax on the worker's labour income decreases the worker's consumption. Hence the entrepreneur decreases the production and thus also the optimal capital stock is decreased.

An increase in the tax on the profit of the entrepreneur, diminishes the incentives for the entrepreneur to produce as much final good as what was optimal before the tax increase. Hence the target level of production will decrease and thus there is no need for as big a capital stock as was the case before the tax change, i.e. the optimal capital stock is decreased.

As mentioned in chapter 3.1, it is incentive incompatible to make a distinction between the investments and the private consumption of the entrepreneurs. Consequently, based on equation (9), also the investments are taxed as consumption. Increasing the tax rate on consumption will increase the entrepreneur's costs of investing in the capital stock. Hence an increased consumption tax rate discourages the entrepreneur from making the optimal investments and consequently the optimal capital stock decreases.

Higher government expenditures increases the demand for goods. Hence the entrepreneur increases the production and thus also the optimal capital stock is increased. However, the increase is very close to zero.

The labour demand decision of the representative entrepreneur is described by the following function:

$$\begin{aligned} & L^*(T^*_{LW}, T^*_{\pi}, T^*_C, G^*), \quad \text{where} \quad \frac{\partial L^*}{\partial T^*_{LW}} < 0, \\ & \frac{\partial L^*}{\partial T^*_{\pi}} < 0, \quad \frac{\partial L^*}{\partial T^*_C} < 0, \quad \frac{\partial L^*}{\partial G^*} > 0. \end{aligned} \tag{22}$$

Because the capital stock decision of the representative entrepreneur is negatively affected by increased tax rates, the entrepreneur will decide to decrease the demand for labour, i.e. less capital involved in the production process results in a lower demand for labour. Higher government expenditures have the opposite effect, however, to a very small extent.

The wage rate decision of the monopoly labour union is described by the following function:

$$\begin{aligned} & w^*(T^*_{LW}, T^*_{\pi}, T^*_C, G^*), \quad \text{where} \quad \frac{\partial w^*}{\partial T^*_{LW}} < 0, \\ & \frac{\partial w^*}{\partial T^*_{\pi}} < 0, \quad \frac{\partial w^*}{\partial T^*_C} < 0, \quad \frac{\partial w^*}{\partial G^*} > 0. \end{aligned} \tag{23}$$

When setting the wage rate, the labour union takes into consideration the subsequent reactions of the entrepreneur. The higher the tax rates are, the higher are the entrepreneur's costs of employing workers. Therefore the union will, in the presence of raised tax rates, decrease the wage rate in order to avoid increasing the pressure on the entrepreneur. Higher government expenditures increase the labour demand and thus also the wages. However, the increase is very close to zero.

The consumption of the representative worker is described by the following function:

$$\begin{aligned} &C^*(T^*_{LW}, T^*_\pi, T^*_C, G^*), \quad \text{where} \quad \frac{\partial C^*}{\partial T^*_{LW}} < 0, \\ &\frac{\partial C^*}{\partial T^*_\pi} > 0, \quad \frac{\partial C^*}{\partial T^*_C} < 0, \quad \frac{\partial C^*}{\partial G^*} < 0. \end{aligned} \tag{24}$$

Increased labour taxes negatively impact the labour demand and the wage rate. Hence these tax increases diminish the possibility to consume.

Even though an increased profit tax results in decreased labour demand and wages, the effect of an increased profit tax on the consumption of the workers is positive. The reason is that the tax increase will result in a lower entrepreneurs' profit, which because of the equilibrium of the goods market will actually increase the workers' consumption.

An increased consumption tax has an indirect as well as a direct negative effect on the worker's consumption. The indirect effect is based on the negative effects on the labour demand and the wage rate. The direct effect is based on the fact that an increased consumption tax increases the worker's consumption cost and hence discourages the worker from consuming.

Public spending crowds out the private consumption of workers, because they both consist of the final good. Hence, it is intuitive that increased government expenditures will negatively affect the representative worker's consumption.

The profit of the representative entrepreneur is described by the following function:

$$\pi^*(T^*_{LW}, T^*_{\pi}, T^*_C, G^*), \quad \text{where} \quad \frac{\partial \pi^*}{\partial T^*_{LW}} < 0,$$
$$\frac{\partial \pi^*}{\partial T^*_{\pi}} < 0, \quad \frac{\partial \pi^*}{\partial T^*_C} > 0, \quad \frac{\partial \pi^*}{\partial G^*} < 0. \tag{25}$$

Increased labour taxes as well as an increased profit tax negatively affect the capital stock as well as the labour demand decision of the representative entrepreneur. The production is based on the capital stock and the labour demand. Therefore these tax increases negatively impact the level of production. Hence the profit declines.

An increased consumption tax affects the entrepreneur's profit in different ways. On one hand, as the investments are taxed as consumption, an increased consumption tax increases the costs of making investments. As a result of this, the entrepreneur decides to invest less. Smaller investments results in a bigger profit. On the other hand, an increased consumption tax, as discussed earlier decreases the capital stock and labour demand of the entrepreneur and consequently also the level of production. This means that the profit decreases. Since the positive effect dominates, the total effect of an increased consumption tax on the profit is positive.

Increased government expenditures affects the entrepreneur's profit in different ways. On one hand, increased expenditures, as discussed earlier slightly increases the capital stock and labour demand of the entrepreneur and consequently also the level of production. This means that the profit increases. On the other hand, increased government expenditures crowd out the entrepreneurs' profits in the goods market. Since the negative effect weakly dominates, the total effect of increased government expenditures on the profit is negative.

The gross income of the representative entrepreneur is described by the following function:

$$\begin{aligned} &Y^*(T^*_{LW}, T^*_{\pi}, T^*_C, G^*), \quad \text{where} \quad \frac{\partial Y^*}{\partial T^*_{LW}} < 0, \\ &\frac{\partial Y^*}{\partial T^*_{\pi}} < 0, \quad \frac{\partial Y^*}{\partial T^*_C} < 0, \quad \frac{\partial Y^*}{\partial G^*} > 0. \end{aligned} \tag{26}$$

Because the capital stock decision as well as the labour demand decision of the representative entrepreneur are negatively affected by increased tax rates, the entrepreneurs' gross income will decrease as a consequence of increased taxes. Higher government expenditures have the opposite effect, however, to a very small extent.

5.2 Tax reforms

The government's budget should in every period be balanced. Consequently, when a tax rate changes, other tax rates and/or the magnitude of the government expenditures must change in order to keep the budget balanced.

Based on the results presented in (21) - (25), it is possible by calibration to find out what the effects of different tax reforms would be.

5.2.1 Tax reform: Shifting the taxation from labour to consumption

In this reform, it is assumed that the profit taxation and the public expenditures are kept constant. Thus when the labour taxation changes, the tax rate on consumption must change in order to keep the government's budget balanced. The calculations needed for estimating the consequences for the households can be found in Appendix D.3.1. The results of such a tax reform are expressed by propositions 1 below.

Proposition 1 *A transfer of taxation from labour to consumption*

- *increases the representative worker's utility*
- *increases the representative entrepreneur's utility*
- *increases the representative worker's consumption*
- *increases the representative entrepreneur's profit*
- *increases the representative entrepreneur's gross income*
- *decreases the wage rate*
- *increases the employment rate*
- *decreases the capital stock*
- *increases the ratio between the representative entrepreneur's profit and the value of the production*
- *decreases the ratio between the workers' labour income and the value of the production*

This reform would generally be beneficial for the society. The wage rate, i.e. the income of an employed worker, is the basis for the unemployment benefit paid to an unemployed worker. Thus the decreased wage rate is negative for the individual worker, employed or unemployed. As anyway the employment rate increases, having a dominating role, the representative worker gains. The underlying reason for this is that the labour taxation is distortional, which effect is increased by the monopoly union's decision making. Compared to this, the consumption taxation is less distortional.

5.2.2 Tax reform: Shifting the taxation from profit to consumption

In this reform, it is assumed that the labour taxation and the public expenditures are kept constant. Thus when the profit taxation changes, the tax rate on consumption must change in order to keep the government's budget balanced. The calculations needed for estimating the consequences for the households can be found in Appendix $D.3.2$. The results of such a tax reform are expressed by propositions 2 below.

Proposition 2 *A transfer of taxation from profit to consumption*

- *decreases the representative worker's utility*
- *increases the representative entrepreneur's utility*
- *decreases the representative worker's consumption*
- *increases the representative entrepreneur's profit*
- *increases the representative entrepreneur's gross income slightly*
- *decreases the wage rate slightly*
- *increases the employment rate slightly*
- *decreases the capital stock*
- *increases the ratio between the representative entrepreneur's profit and the value of the production*
- *decreases the ratio between the workers' labour income and the value of the production slightly*

This reform would hurt the workers and benefit the entrepreneurs. Also in this case the decreased wage rate hurts the individual worker, employed or unemployed. As the increase of the employment rate is close to zero, also the representative worker loses. The underlying reason for this is that the profit taxation has only an indirect effect on the worker, contrary to the consumption taxation. For the entrepreneur the profit taxation is distortional compared to the consumption taxation.

6 Conclusions

This paper discusses possible consequences of certain tax reforms. The system of taxation in the model introduced in this paper consists of taxation on labour, profit and consumption. The effects are measured based on the representative worker's utility and consumption as well as the representative entrepreneur's utility, profit and gross income. Additionally the impacts on the wage rate, the employment rate and the capital stock are examined. Finally the effects on the ratio between the representative entrepreneur's profit and the value of the production as well as the ratio between the workers' labour income and the value of the production are investigated.

Keeping the government's budget balanced, there are two main ways of reforming of the tax system. One is based on the assumption of keeping government expenditures constant and consequently keeping the total tax burden unchanged, shifting the taxation between different types of taxes. The other one includes changing the government expenditures, resulting in a change of the total tax burden. This paper deals with reforms of the first type.

As the government expenditures are not changed, the reforms dealt with in this paper cover decreasing the labour or profit taxation and increasing the consumption taxation. The detailed results of these reforms are presented in chapters 5.2.1 and 5.2.2.

It is shown that the utility increases for workers as well as entrepreneurs when decreasing labour taxes and increasing consumption taxes instead. The effects on the workers' consumption, entrepreneurs' profits and gross income as well as the employment rate are also positive. However, the reform would lead to a decreased wage rate as well as a decreased capital stock. Despite the decreased income of an individual worker, increased employment rate makes the reform beneficial for the representative worker. The reason for these effects is that the labour taxation is distortional, which effect is increased by the monopoly union's decision making. Compared to this, the consumption taxation is less distortional.

It is shown that the utility decreases for workers and increases for the entrepreneurs when decreasing profit taxes and increasing consumption taxes instead. The effects on the entrepreneurs' profits and gross income are positive. However, the reform would lead to a decreased workers' consumption as well as a decreased capital stock. The effect on the wage rate is slightly negative and in this case only slightly positive on the employment rate. The reason for these effects is that the profit taxation has only an indirect effect on the worker, contrary to the consumption taxation. For the entrepreneur the profit taxation is distortional compared to the consumption taxation.

Both reforms shift the income share balance towards the entrepreneurs.

Summarizing the results, this paper shows that shifting to a more consumption-based taxation has a number of positive effects.

A Summary of Parameters

Based on chapter 3, it holds that

$$\begin{aligned}
&\alpha, \eta, \sigma \in (0,1), \quad \psi, \rho, P_t > 0, \quad \delta \in [0,1], \quad r_t \in (-\infty, \infty) \\
&T_{LW,t}, T_{\pi,t}, T_{C,t} \in (-\infty, 1).
\end{aligned} \tag{27}$$

B Validity of Walras' law

When checking if Walras' law holds, one needs to consider the budget constraint of the workers, the profits of the entrepreneurs as well as the budget constraint of the government.

The budget constraint of the workers, according to equation (3), is

$$\begin{aligned}
&(1+r_t)W_t + L_t w_t(1-T_{LW,t}) + (N-L_t)\alpha w_t(1-T_{LW,t}) \\
&= (1+T_{C,t})P_t C_t + W_{t+1},
\end{aligned} \tag{28}$$

The profits of the entrepreneurs, based on equations (9) and (11), is

$$\begin{aligned}
&-\pi_t + (1-T_{\pi,t})\big[P_t F(K_t, L_t) - w_t L_t \\
&- (1+T_{C,t})P_t\big(K_{t+1} - (1-\delta)K_t\big)\big] = 0.
\end{aligned} \tag{29}$$

The budget constraint of the government, according to equation (16), is

$$\begin{aligned}
&T_{\pi,t}\big[P_t F(K_t, L_t) - w_t L_t - (1+T_{C,t})P_t\big(K_{t+1} - (1-\delta)K_t\big)\big] \\
&+ T_{C,t}P_t\big(C_t + K_{t+1} - (1-\delta)K_t\big) + T_{LW,t}w_t L_t \\
&+ D_{t+1} - (1+r_t)D_t - P_t G_t - (N-L_t)\alpha w_t(1-T_{LW,t}) = 0.
\end{aligned} \tag{30}$$

Based on equation (17), equation (30) can be rewritten as

$$\begin{aligned}
&T_{\pi,t}\big[P_t F(K_t, L_t) - w_t L_t - (1+T_{C,t})P_t\big(K_{t+1} - (1-\delta)K_t\big)\big] \\
&+ T_{C,t}P_t\big(C_t + K_{t+1} - (1-\delta)K_t\big) + T_{LW,t}w_t L_t \\
&+ W_{t+1} - (1+r_t)W_t - P_t G_t - (N-L_t)\alpha w_t(1-T_{LW,t}) = 0.
\end{aligned} \tag{31}$$

Adding equations (28), (29) and (31) leads to

$$
\begin{aligned}
&(1+r_t)W_t + L_t w_t(1-T_{LW,t}) + (N-L_t)\alpha w_t(1-T_{LW,t}) \\
&+ T_{\pi,t}\big[P_tF(K_t,L_t) - w_tL_t - (1+T_{C,t})P_t\big(K_{t+1}-(1-\delta)K_t\big)\big] \\
&+ T_{C,t}P_t\big(C_t + K_{t+1} - (1-\delta)K_t\big) + T_{LW,t}w_tL_t \\
&+ W_{t+1} - (1+r_t)W_t - P_tG_t - (N-L_t)\alpha w_t(1-T_{LW,t}) \\
&- \pi_t + (1-T_{\pi,t})\big[P_tF(K_t,L_t) - w_tL_t \\
&- (1+T_{C,t})P_t\big(K_{t+1}-(1-\delta)K_t\big)\big] = (1+T_{C,t})P_tC_t + W_{t+1}
\end{aligned}
$$

$$
\begin{aligned}
&\Leftrightarrow -P_tG_t - \pi_t + P_tF(K_t,L_t) - P_t\big(K_{t+1}-(1-\delta)K_t\big) = P_tC_t \\
&\Leftrightarrow P_tF(K_t,L_t) = P_tC_t + P_tG_t + \pi_t + P_t\big(K_{t+1}-(1-\delta)K_t\big),
\end{aligned}
\tag{32}
$$

which describes the general goods market balance outside the steady state.

C Solving the Model

Based on the description in chapter 3, the model is now solved step-by-step.

C.1 Workers

The state variables in period t are $T_{LW,t}$, $T_{C,t}$, r_t, W_t and w_t. Based on (2) and (4), the maximization problem becomes

$$
\begin{aligned}
&\mathcal{V}_t = \mathcal{V}(T_{LW,t}, T_{C,t}, r_t, W_t, w_t) = \max_{W_{t+1}}\Big\{\log(C_t) + \psi\log(N-L_t) \\
&+ \beta\mathcal{V}(T_{LW,t+1}, T_{C,t+1}, r_{t+1}, W_{t+1}, w_{t+1})\Big\}, \\
&\text{where} \quad C_t = \frac{(1+r_t)W_t}{P_t(1+T_{C,t})} + \frac{L_tw_t(1-T_{LW,t})}{P_t(1+T_{C,t})} \\
&+ \frac{(N-L_t)\alpha w_t(1-T_{LW,t})}{P_t(1+T_{C,t})} - \frac{W_{t+1}}{P_t(1+T_{C,t})}
\end{aligned}
\tag{33}
$$

with $\beta = \frac{1}{1+\rho}$ being the discount factor and ρ the rate of time preference.

Maximization with respect to W_{t+1} leads to

$$\frac{1}{C_t} \times (-\frac{1}{P_t(1+T_{C,t})}) + \beta \frac{\partial \mathcal{V}_{t+1}}{\partial W_{t+1}} = 0. \tag{34}$$

Based on the first-order condition (34), the saved assets for period t+1 are dependent on the saved assets for period t, i.e. the saved assets for period t+1 can be expressed as the decision rule $W_{t+1}(W_t)$. Now the value function (33) is differentiated with respect to W_t, keeping the savings function $W_{t+1}(W_t)$ in mind.

$$\begin{aligned}
\frac{\partial \mathcal{V}_t}{\partial W_t} &= \frac{1}{C_t}\left[\frac{1+r_t}{P_t(1+T_{C,t})} - \frac{1}{P_t(1+T_{C,t})}\frac{\partial W_{t+1}}{\partial W_t}\right] + \beta \frac{\partial \mathcal{V}_{t+1}}{\partial W_{t+1}}\frac{\partial W_{t+1}}{\partial W_t} \\
&= \frac{1}{C_t} \times \frac{1+r_t}{P_t(1+T_{C,t})} + \left[\frac{1}{C_t} \times (-\frac{1}{P_t(1+T_{C,t})}) + \beta \frac{\partial \mathcal{V}_{t+1}}{\partial W_{t+1}}\right]\frac{\partial W_{t+1}}{\partial W_t} \\
&\Leftrightarrow \frac{\partial \mathcal{V}_t}{\partial W_t} = \frac{1}{C_t} \times \frac{1+r_t}{P_t(1+T_{C,t})}. \qquad (35)
\end{aligned}$$

Forwarding equation (35) by one period generates

$$\frac{\partial \mathcal{V}_{t+1}}{\partial W_{t+1}} = \frac{1}{C_{t+1}} \times \frac{1+r_{t+1}}{P_{t+1}(1+T_{C,t+1})}$$

and substituting the received equation into the first-order condition (34) leads to

$$\begin{aligned}
&\frac{1}{C_t} \times (-\frac{1}{P_t(1+T_{C,t})}) + \beta \frac{1}{C_{t+1}} \times \frac{1+r_{t+1}}{P_{t+1}(1+T_{C,t+1})} = 0 \\
&\Leftrightarrow \frac{1}{C_t} \times \frac{1}{P_t(1+T_{C,t})} = \beta \frac{1}{C_{t+1}} \times \frac{1+r_{t+1}}{P_{t+1}(1+T_{C,t+1})} \\
&\Leftrightarrow \frac{C_{t+1}}{C_t} = \beta \frac{P_t(1+T_{C,t})}{P_{t+1}(1+T_{C,t+1})}(1+r_{t+1}). \qquad (36)
\end{aligned}$$

Equation (36) is the Euler equation for the representative worker.

C.2 Entrepreneurs

The state variables in period t are $T_{\pi,t}$, $T_{C,t}$, K_t and w_t. Based on (7) and (11), the maximization problem takes the following form:

$$\begin{aligned} &\mathcal{V}_{t,f} = \mathcal{V}(T_{\pi,t}, T_{C,t}, K_t, w_t) = \\ &\max_{K_{t+1},L_t} \left\{ \log(\pi_t) + \frac{1}{1+r_{t+1}} \mathcal{V}(T_{\pi,t+1}, T_{C,t+1}, K_{t+1}, w_{t+1}) \right\}, \\ &\text{where} \quad \pi_t = (1-T_{\pi,t})\big[P_t F(K_t, L_t) - w_t L_t \\ &-(1+T_{C,t})P_t\big(K_{t+1} - (1-\delta)K_t\big)\big]. \end{aligned} \tag{37}$$

The production function $F(K_t, L_t)$ is expressed by formula (12).

Maximization with respect to K_{t+1} now generates

$$-\frac{(1-T_{\pi,t})(1+T_{C,t})P_t}{\pi_t} + \frac{1}{1+r_{t+1}} \frac{\partial \mathcal{V}_{t+1,f}}{\partial K_{t+1}} = 0. \tag{38}$$

Based on the first-order condition (38), the capital stock in period t+1 is dependent on the capital stock in period t, i.e. the capital stock in period t+1 can be expressed as the decision rule $K_{t+1}(K_t)$. Now the value function (37) is differentiated with respect to K_t, keeping the decision rule in mind.

$$\begin{aligned} &\frac{\partial \mathcal{V}_{t,f}}{\partial K_t} = \frac{(1-T_{\pi,t})}{\pi_t}\left[P_t \frac{\partial F(K_t, L_t)}{\partial K_t} - (1+T_{C,t})P_t\left(\frac{\partial K_{t+1}}{\partial K_t} - 1 + \delta\right)\right] \\ &+ \frac{1}{1+r_{t+1}} \frac{\partial \mathcal{V}_{t+1,f}}{\partial K_{t+1}} \frac{\partial K_{t+1}}{\partial K_t} = \frac{(1-T_{\pi,t})P_t}{\pi_t}\left[F_K(K_t, L_t) + (1+T_{C,t})(1-\delta)\right] \\ &+ \left[-\frac{(1-T_{\pi,t})(1+T_{C,t})P_t}{\pi_t} + \frac{1}{1+r_{t+1}} \frac{\partial \mathcal{V}_{t+1,f}}{\partial K_{t+1}}\right] \frac{\partial K_{t+1}}{\partial K_t} \end{aligned}$$

$$\Leftrightarrow \frac{\partial \mathcal{V}_{t,f}}{\partial K_t} = \frac{(1-T_{\pi,t})P_t}{\pi_t}\left[F_K(K_t, L_t) + (1+T_{C,t})(1-\delta)\right]. \tag{39}$$

Forwarding equation (39) by one period generates

$$\Leftrightarrow \frac{\partial \mathcal{V}_{t+1,f}}{\partial K_{t+1}} = \frac{(1-T_{\pi,t+1})P_{t+1}}{\pi_{t+1}}\left[F_K(K_{t+1}, L_{t+1}) + (1+T_{C,t+1})(1-\delta)\right]$$

and substituting the received equation into the first-order condition (38) leads to

$$
\begin{aligned}
&-\frac{(1-T_{\pi,t})(1+T_{C,t})P_t}{\pi_t}+\frac{1}{1+r_{t+1}}\times\frac{(1-T_{\pi,t+1})P_{t+1}}{\pi_{t+1}} \\
&\times\big[F_K(K_{t+1},L_{t+1})+(1+T_{C,t+1})(1-\delta)\big]=0.
\end{aligned}
\tag{40}
$$

Based on production function (12), the expression for $F_K(K_t, L_t)$ in equation (40) is received from the following calculation:

$$
\begin{aligned}
F_K(K_t,L_t)&=A\frac{\sigma}{\sigma-1}[\eta K_t^{\frac{\sigma-1}{\sigma}}+(1-\eta)L_t^{\frac{\sigma-1}{\sigma}}]^{\frac{1}{\sigma-1}}\eta\frac{\sigma-1}{\sigma}K_t^{-\frac{1}{\sigma}} \\
\Leftrightarrow F_K(K_t,L_t)&=A\eta[\eta K_t^{\frac{\sigma-1}{\sigma}}+(1-\eta)L_t^{\frac{\sigma-1}{\sigma}}]^{\frac{1}{\sigma-1}}K_t^{-\frac{1}{\sigma}}.
\end{aligned}
\tag{41}
$$

Now totally differentiating equation (40) with respect to K_{t+1}, K_t, w_t, P_t, $T_{\pi,t}$ and $T_{C,t}$, considering equation (11), generates

$$
\begin{aligned}
&\Big\{-\frac{(1-T_{\pi,t})^2(1+T_{C,t})^2P_t^2}{\pi_t^2}-\frac{(1-T_{\pi,t+1})^2P_{t+1}^2}{(1+r_{t+1})\pi_{t+1}^2}\\
&\times\big[F_K(K_{t+1},L_{t+1})+(1+T_{C,t+1})(1-\delta)\big]^2\\
&+\frac{(1-T_{\pi,t+1})P_{t+1}F_{KK}(K_{t+1},L_{t+1})}{(1+r_{t+1})\pi_{t+1}}\Big\}dK_{t+1}\\
&+\Big[\frac{(1-T_{\pi,t})^2(1+T_{C,t})P_t^2}{\pi_t^2}\big(F_K(K_t,L_t)+(1+T_{C,t})(1-\delta)\big)\Big]dK_t\\
&-\frac{(1-T_{\pi,t})^2(1+T_{C,t})P_t(1+T_{LE,t})L_t}{\pi_t^2}dw_t\\
&\Big[-\frac{(1-T_{\pi,t})(1+T_{C,t})}{\pi_t}+\frac{(1-T_{\pi,t})^2(1+T_{C,t})P_t}{\pi_t^2}\\
&\times\big[F(K_t,L_t)-(1+T_{C,t})\big(K_{t+1}-(1-\delta)K_t\big)\big]\Big]dP_t\\
&+\Big[\frac{(1+T_{C,t})P_t}{\pi_t}-\frac{(1-T_{\pi,t})(1+T_{C,t})P_t}{\pi_t^2}\\
&\times\big[P_tF(K_t,L_t)-(1+T_{LE,t})w_tL_t\\
&-(1+T_{C,t})P_t\big(K_{t+1}-(1-\delta)K_t\big)\big]\Big]dT_{\pi,t}+\Big[-\frac{(1-T_{\pi,t})P_t}{\pi_t}\\
&-\frac{(1-T_{\pi,t})^2(1+T_{C,t})P_t^2\big(K_{t+1}-(1-\delta)K_t\big)}{\pi_t^2}\Big]dT_{C,t}=0.
\end{aligned}
\tag{42}
$$

Based on equation (42), the capital choice of the representative entrepreneur can be expressed as

$$
K_{t+1}(K_t,w_t,P_t,T_{\pi,t},T_{C,t}) \tag{43}
$$

Based on equation (41), the expression for $F_{KK}(K_t,L_t)$ in equation (42) can be received from the following calculation:

$$
\begin{aligned}
&F_{KK}(K_t,L_t)=A\eta\frac{1}{\sigma-1}[\eta K_t^{\frac{\sigma-1}{\sigma}}+(1-\eta)L_t^{\frac{\sigma-1}{\sigma}}]^{\frac{2-\sigma}{\sigma-1}}\eta\frac{\sigma-1}{\sigma}K_t^{-\frac{1}{\sigma}}K_t^{-\frac{1}{\sigma}}\\
&+A\eta[\eta K_t^{\frac{\sigma-1}{\sigma}}+(1-\eta)L_t^{\frac{\sigma-1}{\sigma}}]^{\frac{1}{\sigma-1}}(-\frac{1}{\sigma})K_t^{-\frac{\sigma+1}{\sigma}}
\end{aligned}
$$

$$
\begin{aligned}
&\Leftrightarrow F_{KK}(K_t,L_t)=A\frac{\eta^2}{\sigma}[\eta K_t^{\frac{\sigma-1}{\sigma}}+(1-\eta)L_t^{\frac{\sigma-1}{\sigma}}]^{\frac{2-\sigma}{\sigma-1}}K_t^{-\frac{2}{\sigma}}\\
&-A\frac{\eta}{\sigma}[\eta K_t^{\frac{\sigma-1}{\sigma}}+(1-\eta)L_t^{\frac{\sigma-1}{\sigma}}]^{\frac{1}{\sigma-1}}K_t^{-\frac{\sigma+1}{\sigma}}
\end{aligned}
\tag{44}
$$

Regarding the optimal labour demand, i.e. the labour demand L_t that maximizes the value function specified in (37), it holds that

$$L_t = \arg\max_{L_t} \left\{ \log(\pi_t) + \frac{1}{1+r_{t+1}} \mathcal{V}(T_{\pi,t+1}, T_{C,t+1}, K_{t+1}, w_{t+1}) \right\}$$
$$= \arg\max_{L_t} \quad \log(\pi_t) = \arg\max_{L_t} \quad \pi_t.$$

Consequently the maximization problem (37) can now be rewritten as

$$\begin{aligned} \max_{L_t} \Big\{ & (1 - T_{\pi,t}) \big[P_t F(K_t, L_t) - w_t L_t \\ & - (1 + T_{C,t}) P_t \big(K_{t+1} - (1-\delta) K_t \big) \big] \Big\}. \end{aligned} \tag{45}$$

Maximization of (37) with respect to L_t generates:

$$\begin{aligned} & (1 - T_{\pi,t}) \left(P_t \frac{\partial F(K_t, L_t)}{\partial L_t} - w_t \right) = 0 \\ & \Leftrightarrow (1 - T_{\pi,t}) \big(P_t F_L(K_t, L_t) - w_t \big) = 0, \end{aligned} \tag{46}$$

which, based on (27), can be rewritten as

$$P_t F_L(K_t, L_t) - w_t = 0. \tag{47}$$

Based on production function (12), the expression for $F_L(K_t, L_t)$ in equation (47) is received from the following calculation:

$$F_L(K_t, L_t) = A \frac{\sigma}{\sigma - 1} [\eta K_t^{\frac{\sigma-1}{\sigma}} + (1-\eta) L_t^{\frac{\sigma-1}{\sigma}}]^{\frac{1}{\sigma-1}} (1-\eta) \frac{\sigma - 1}{\sigma} L_t^{-\frac{1}{\sigma}}$$
$$\Leftrightarrow F_L(K_t, L_t) = A(1-\eta) [\eta K_t^{\frac{\sigma-1}{\sigma}} + (1-\eta) L_t^{\frac{\sigma-1}{\sigma}}]^{\frac{1}{\sigma-1}} L_t^{-\frac{1}{\sigma}}. \tag{48}$$

Differentiating totally equation (47) with respect to L_t, K_t, w_t and P_t generates

$$P_tF_{LL}(K_t,L_t)dL_t + P_tF_{LK}(K_t,L_t)dK_t - dw_t + F_L(K_t,L_t)dP_t = 0. \tag{49}$$

With $dw_t = dP_t = 0$, equation (49) can be rewritten as

$$\frac{\partial L_t}{\partial K_t} = -\frac{F_{LK}(K_t,L_t)}{F_{LL}(K_t,L_t)}. \tag{50}$$

With $dK_t = dP_t = 0$, equation (49) can be rewritten as

$$\frac{\partial L_t}{\partial w_t} = \frac{1}{P_tF_{LL}(K_t,L_t)}. \tag{51}$$

With $dK_t = dw_t = 0$, equation (49) can be rewritten as

$$\frac{\partial L_t}{\partial P_t} = -\frac{F_L(K_t,L_t)}{P_tF_{LL}(K_t,L_t)}. \tag{52}$$

Based on equations (50) - (52), the labour demand of the representative entrepreneur can be expressed as

$$L_t(K_t,w_t,P_t) \tag{53}$$

Based on equation (48), the expression for $F_{LL}(K_t,L_t)$ in equations (49) - (**??**) can be received from the following calculation:

$$F_{LL}(K_t,L_t) = A(1-\eta)\frac{1}{\sigma-1}[\eta K_t^{\frac{\sigma-1}{\sigma}} + (1-\eta)L_t^{\frac{\sigma-1}{\sigma}}]^{\frac{2-\sigma}{\sigma-1}}(1-\eta)$$
$$\times \frac{\sigma-1}{\sigma}L_t^{-\frac{1}{\sigma}}L_t^{-\frac{1}{\sigma}} + A(1-\eta)[\eta K_t^{\frac{\sigma-1}{\sigma}} + (1-\eta)L_t^{\frac{\sigma-1}{\sigma}}]^{\frac{1}{\sigma-1}}(-\frac{1}{\sigma})L_t^{-\frac{\sigma+1}{\sigma}}$$

$$
\begin{aligned}
&\Leftrightarrow F_{LL}(K_t, L_t) = A\frac{(1-\eta)^2}{\sigma}[\eta K_t^{\frac{\sigma-1}{\sigma}} + (1-\eta)L_t^{\frac{\sigma-1}{\sigma}}]^{\frac{2-\sigma}{\sigma-1}} L_t^{-\frac{2}{\sigma}} \\
&- A\frac{(1-\eta)}{\sigma}[\eta K_t^{\frac{\sigma-1}{\sigma}} + (1-\eta)L_t^{\frac{\sigma-1}{\sigma}}]^{\frac{1}{\sigma-1}} L_t^{-\frac{\sigma+1}{\sigma}}.
\end{aligned} \tag{54}
$$

Based on equation (48), the expression for $F_{LK}(K_t, L_t)$ in equation (50) can be received from the following calculation:

$$
F_{LK}(K_t, L_t) = A(1-\eta)\frac{1}{\sigma-1}[\eta K_t^{\frac{\sigma-1}{\sigma}} + (1-\eta)L_t^{\frac{\sigma-1}{\sigma}}]^{\frac{2-\sigma}{\sigma-1}} \eta \frac{\sigma-1}{\sigma} K_t^{-\frac{1}{\sigma}} L_t^{-\frac{1}{\sigma}}
$$

$$
\Leftrightarrow F_{LK}(K_t, L_t) = A\frac{(1-\eta)\eta}{\sigma}[\eta K_t^{\frac{\sigma-1}{\sigma}} + (1-\eta)L_t^{\frac{\sigma-1}{\sigma}}]^{\frac{2-\sigma}{\sigma-1}} K_t^{-\frac{1}{\sigma}} L_t^{-\frac{1}{\sigma}}. \tag{55}
$$

C.3 Labour union

The state variables in period t are $T_{LW,t}$, $T_{C,t}$, r_t and W_t. Based on (2) and (4), the maximization problem becomes

$$
\begin{aligned}
&\mathcal{V}_{t,U} = \mathcal{V}(T_{LW,t}, T_{C,t}, r_t, W_t) = \max_{w_t}\Big\{\log(C_t) + \psi\log(N - L_t) \\
&+ \beta\mathcal{V}(T_{LW,t+1}, T_{C,t+1}, r_{t+1}, W_{t+1}, w_{t+1})\Big\}, \\
&\text{where} \quad C_t = \frac{(1+r_t)W_t}{P_t(1+T_{C,t})} + \frac{L_t w_t(1-T_{LW,t})}{P_t(1+T_{C,t})} \\
&+ \frac{(N-L_t)\alpha w_t(1-T_{LW,t})}{P_t(1+T_{C,t})} - \frac{W_{t+1}}{P_t(1+T_{C,t})},
\end{aligned} \tag{56}
$$

with $\beta = \frac{1}{1+\rho}$ being the discount factor and ρ the rate of time preference.

Maximization with respect to w_t, taking into consideration equation (51), leads to

$$
\begin{aligned}
&\frac{1}{C_t}\left[\frac{\big(L_t+(N-L_t)\alpha\big)(1-T_{LW,t})}{P_t(1+T_{C,t})}+\frac{\partial L_t}{\partial w_t}\times\frac{(1-\alpha)w_t(1-T_{LW,t})}{P_t(1+T_{C,t})}\right]\\
&-\psi\frac{1}{N-L_t}\frac{\partial L_t}{\partial w_t}=0\\
&\Leftrightarrow\frac{1}{C_t}\left[\frac{\big(L_t+(N-L_t)\alpha\big)(1-T_{LW,t})}{P_t(1+T_{C,t})}\right.\\
&\left.+\frac{(1-\alpha)w_t(1-T_{LW,t})}{P_t^2F_{LL}(K_t,L_t)(1+T_{C,t})}\right]-\psi\frac{1}{(N-L_t)P_tF_{LL}(K_t,L_t)}=0
\end{aligned}
$$

$$
\begin{aligned}
&\Leftrightarrow\big(L_t+(N-L_t)\alpha\big)(1-T_{LW,t})(N-L_t)P_tF_{LL}(K_t,L_t)\\
&+(1-\alpha)(N-L_t)w_t(1-T_{LW,t})-\psi(1+T_{C,t})P_tC_t=0.
\end{aligned}
\tag{57}
$$

D Comparative statics

The comparative statistics analysis will here be done by first collecting the steady state equations, representing the optimal choices of individuals, firms and the union, and then applying Cramer's rule on the received system of equations.

The production function $F(K_t, L_t)$ is assumed to be approximated using a 2nd degree Taylor approximation polynomial:

$$
\begin{aligned}
&F(K_t,L_t)\approx\\
&F(K^*,L^*)+F_K(K^*,L^*)(K_t-K^*)+F_L(K^*,L^*)(L_t-L^*)\\
&+\frac{1}{2}\Big[F_{KK}(K^*,L^*)(K_t-K^*)^2+F_{KL}(K^*,L^*)(L_t-L^*)^2\\
&+F_{LK}(K^*,L^*)(K_t-K^*)^2+F_{LL}(K^*,L^*)(L_t-L^*)^2\Big].
\end{aligned}
\tag{58}
$$

This shows that any 3rd or higher degree derivative of the production function must equal zero.

D.1 Steady state equations

In this section, the following budget constraints and optimality conditions will be transformed into their steady state counterparts: (3), (5), (11), (13), (14) and (15). In steady state all tax rates, being exogenous variables, will be constant.

Based on equation (3), the steady state budget for the representative worker is expressed by

$$\begin{aligned}&(1+r^*)W^* + L^*w^*(1-T^*_{LW}) + (N-L^*)\alpha w^*(1-T^*_{LW})\\&= (1+T^*_C)P^*C^* + W^*\\&\quad\Leftrightarrow r^*W^* + L^*w^*(1-T^*_{LW}) + (N-L^*)\alpha w^*(1-T^*_{LW})\\&\quad-(1+T^*_C)P^*C^* = 0.\end{aligned} \tag{59}$$

Applying steady state values to the Euler equation for the representative worker, i.e. equation (5) and taking in consideration that $\beta = \frac{1}{1+\rho}$ the following calculation is received

$$\begin{aligned}&\frac{C^*}{C^*} = \beta\frac{P^*(1+T^*_C)}{P^*(1+T^*_C)}(1+r^*) \Leftrightarrow 1 = \frac{1}{1+\rho}(1+r^*)\\&\Leftrightarrow \rho = r^*,\end{aligned} \tag{60}$$

which expresses the equilibrium steady state interest rate.

Based on equation (11), the steady state profit for the representative entrepreneur can be received from the following calculation, taking in consideration equations (9) and (12):

$$\begin{aligned}&\pi_t = (1-T_{\pi,t})\big[P_tF(K_t,L_t) - w_tL_t - (1+T_{C,t})P_t\big(K_{t+1}-(1-\delta)K_t\big)\big]\\&\Leftrightarrow \pi_t = (1-T_{\pi,t})\big[P_tA[\eta K_t^{\frac{\sigma-1}{\sigma}} + (1-\eta)L_t^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} - w_tL_t\\&-(1+T_{C,t})P_t\big(K_{t+1}-(1-\delta)K_t\big)\big]\\&\Leftrightarrow \pi^* = (1-T^*_\pi)\Big[P^*A[\eta(K^*)^{\frac{\sigma-1}{\sigma}} + (1-\eta)(L^*)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}\\&-w^*L^* - (1+T^*_C)P^*\big(K^*-(1-\delta)K^*\big)\Big]\end{aligned}$$

$$\Leftrightarrow -\pi^* + (1 - T_\pi^*)\Big[P^* A[\eta(K^*)^{\frac{\sigma-1}{\sigma}} + (1-\eta)(L^*)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}$$
$$- w^* L^* - (1 + T_C^*)P^* \delta K^*\Big] = 0. \tag{61}$$

The entrepreneur's optimality condition for the capital stock (13) in steady state is, taking in consideration (27) and (41), reached by the calculation below:

$$-\frac{(1 - T_{\pi,t})(1 + T_{C,t})P_t}{\pi_t} + \frac{1}{1 + r_{t+1}} \times \frac{(1 - T_{\pi,t+1})P_{t+1}}{\pi_{t+1}}$$
$$\times \Big[F_K(K_{t+1}, L_{t+1}) + (1 + T_{C,t+1})(1 - \delta)\Big] = 0$$

$$\Leftrightarrow -\frac{(1 - T_\pi^*)(1 + T_C^*)P^*}{\pi^*} + \frac{1}{1 + r^*} \times \frac{(1 - T_\pi^*)P^*}{\pi^*}$$
$$\times \Big[F_K(K^*, L^*) + (1 + T_C^*)(1 - \delta)\Big] = 0$$

$$\Leftrightarrow -(1 + r^*)(1 + T_C^*) + F_K(K^*, L^*) + (1 + T_C^*)(1 - \delta) = 0$$
$$\Leftrightarrow F_K(K^*, L^*) - (1 + T_C^*)(1 + r^* - 1 + \delta) = 0$$

$$\Leftrightarrow A\eta[\eta(K^*)^{\frac{\sigma-1}{\sigma}} + (1-\eta)(L^*)^{\frac{\sigma-1}{\sigma}}]^{\frac{1}{\sigma-1}}(K^*)^{-\frac{1}{\sigma}}$$
$$- (1 + T_C^*)\big(r^* + \delta\big) = 0. \tag{62}$$

The entrepreneur's optimality condition for the labour demand (14) in steady state is, taking in consideration equation (48), reached by the calculation below

$$P^* F_L(K^*, L^*) - w^* = 0$$

$$\Leftrightarrow P^* A(1-\eta)[\eta(K^*)^{\frac{\sigma-1}{\sigma}} + (1-\eta)(L^*)^{\frac{\sigma-1}{\sigma}}]^{\frac{1}{\sigma-1}}(L^*)^{-\frac{1}{\sigma}} - w^* = 0. \tag{63}$$

The labour union's optimality condition for the wage rate (15) in steady state is, taking in consideration equations (54), reached by the calculation below

$$
\begin{aligned}
&\big(L^* + (N - L^*)\alpha\big)(1 - T^*_{LW})(N - L^*)P^* F_{LL}(K^*, L^*) \\
&+ (1 - \alpha)(N - L^*)w^*(1 - T^*_{LW}) - \psi(1 + T^*_C)P^* C^* = 0 \\
&\Leftrightarrow \big(L^* + (N - L^*)\alpha\big)(1 - T^*_{LW})(N - L^*)P^* \\
&\times \Bigg(A\frac{(1-\eta)^2}{\sigma}[\eta(K^*)^{\frac{\sigma-1}{\sigma}} + (1-\eta)(L^*)^{\frac{\sigma-1}{\sigma}}]^{\frac{2-\sigma}{\sigma-1}}(L^*)^{-\frac{2}{\sigma}} \\
&- A\frac{(1-\eta)}{\sigma}[\eta(K^*)^{\frac{\sigma-1}{\sigma}} + (1-\eta)(L^*)^{\frac{\sigma-1}{\sigma}}]^{\frac{1}{\sigma-1}}(L^*)^{-\frac{\sigma+1}{\sigma}} \Bigg) \\
&+ (1 - \alpha)(N - L^*)w^*(1 - T^*_{LW}) - \psi(1 + T^*_C)P^* C^* = 0.
\end{aligned}
\tag{64}
$$

In the steady state it holds that the goods market is balanced. This means that the steady state production $F(K^*, L^*)$ equals the sum of the representative worker's consumption C^*, the government expenditures G^*, the representative entrepreneur's net profit π^* and the steady state investments I^* including the adjustment costs. Consequently, based on equation (18) and taking in consideration equations (9) and (12), it holds that

$$
P^* F(K^*, L^*) = P^* C^* + P^* G^* + \pi^* + P^*\big(K^* - (1 - \delta)K^*\big)
$$

$$
\begin{aligned}
&\Leftrightarrow P^* A\big[\eta(K^*)^{\frac{\sigma-1}{\sigma}} + (1-\eta)(L^*)^{\frac{\sigma-1}{\sigma}}\big]^{\frac{\sigma}{\sigma-1}} - P^* C^* \\
&- P^* G^* - \pi^* - P^*\delta K^* = 0.
\end{aligned}
\tag{65}
$$

Based on the equations (59) - (65), the following system of steady state equations is received:

$$
\begin{cases}
\rho W^* + L^* w^*(1-T^*_{LW}) + (N-L^*)\alpha w^*(1-T^*_{LW}) - (1+T^*_C)P^*C^* = 0 \quad \doteq \quad f^1 \\
-\pi^* + (1-T^*_\pi)\Big[P^*A[\eta(K^*)^{\frac{\sigma-1}{\sigma}} + (1-\eta)(L^*)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} \\
-w^*L^* - (1+T^*_C)P^*\delta K^*\Big] = 0 \quad \doteq \quad f^2 \\
A\eta[\eta(K^*)^{\frac{\sigma-1}{\sigma}} + (1-\eta)(L^*)^{\frac{\sigma-1}{\sigma}}]^{\frac{1}{\sigma-1}}(K^*)^{-\frac{1}{\sigma}} - (1+T^*_C)\big(\rho+\delta\big) = 0 \quad \doteq \quad f^3 \\
P^*A(1-\eta)[\eta(K^*)^{\frac{\sigma-1}{\sigma}} + (1-\eta)(L^*)^{\frac{\sigma-1}{\sigma}}]^{\frac{1}{\sigma-1}}(L^*)^{-\frac{1}{\sigma}} - w^* = 0 \quad \doteq \quad f^4 \\
\big(L^* + (N-L^*)\alpha\big)(1-T^*_{LW})(N-L^*)P^* \\
\times\Big(A\frac{(1-\eta)^2}{\sigma}[\eta(K^*)^{\frac{\sigma-1}{\sigma}} + (1-\eta)(L^*)^{\frac{\sigma-1}{\sigma}}]^{\frac{2-\sigma}{\sigma-1}}(L^*)^{-\frac{2}{\sigma}} \\
-A\frac{(1-\eta)}{\sigma}[\eta(K^*)^{\frac{\sigma-1}{\sigma}} + (1-\eta)(L^*)^{\frac{\sigma-1}{\sigma}}]^{\frac{1}{\sigma-1}}(L^*)^{-\frac{\sigma+1}{\sigma}}\Big) \\
+(1-\alpha)(N-L^*)w^*(1-T^*_{LW}) - \psi(1+T^*_C)P^*C^* = 0 \quad \doteq \quad f^5 \\
P^*A\big[\eta(K^*)^{\frac{\sigma-1}{\sigma}} + (1-\eta)(L^*)^{\frac{\sigma-1}{\sigma}}\big]^{\frac{\sigma}{\sigma-1}} - P^*C^* - P^*G^* - \pi^* - P^*\delta K^* = 0 \quad \doteq \quad f^6
\end{cases}
$$

where the endogenous variables K^*, L^*, w^*, C^*, π^* and W^* form the following vector

$$
EN = \begin{bmatrix} K^* \\ L^* \\ w^* \\ C^* \\ \pi^* \\ W^* \end{bmatrix}
$$

and where the exogenous variables T^*_{LW}, T^*_π, T^*_C and G^* form the following vector

$$
EX = \begin{bmatrix} T^*_{LW} \\ T^*_\pi \\ T^*_C \\ G^* \end{bmatrix}.
$$

D.2 Application of Cramer's rule

The system of equations expressed in the previous section is now used in order to apply Cramer's rule, i.e.

$$\begin{bmatrix} f_K^1 & f_L^1 & f_w^1 & f_C^1 & f_\pi^1 & f_W^1 \\ f_K^2 & f_L^2 & f_w^2 & f_C^2 & f_\pi^2 & f_W^2 \\ f_K^3 & f_L^3 & f_w^3 & f_C^3 & f_\pi^3 & f_W^3 \\ f_K^4 & f_L^4 & f_w^4 & f_C^4 & f_\pi^4 & f_W^4 \\ f_K^5 & f_L^5 & f_w^5 & f_C^5 & f_\pi^5 & f_W^5 \\ f_K^6 & f_L^6 & f_w^6 & f_C^6 & f_\pi^6 & f_W^6 \end{bmatrix} \begin{bmatrix} dK^* \\ dL^* \\ dw^* \\ dC^* \\ d\pi^* \\ dW^* \end{bmatrix}$$

$$+ \begin{bmatrix} f_{T_{LW}}^1 & f_{T_\pi}^1 & f_{T_C}^1 & f_G^1 \\ f_{T_{LW}}^2 & f_{T_\pi}^2 & f_{T_C}^2 & f_G^2 \\ f_{T_{LW}}^3 & f_{T_\pi}^3 & f_{T_C}^3 & f_G^3 \\ f_{T_{LW}}^4 & f_{T_\pi}^4 & f_{T_C}^4 & f_G^4 \\ f_{T_{LW}}^5 & f_{T_\pi}^5 & f_{T_C}^5 & f_G^5 \\ f_{T_{LW}}^6 & f_{T_\pi}^6 & f_{T_C}^6 & f_G^6 \end{bmatrix} \begin{bmatrix} dT_{LW}^* \\ dT_\pi^* \\ dT_C^* \\ dG^* \end{bmatrix} = 0$$

For an endogenous variable X^*, the Cramer's rule will generate the following equation:

$$dX^* = -\frac{|D_{X,T_{LW}}|}{|D|} dT_{LW}^* - \frac{|D_{X,T_\pi}|}{|D|} dT_\pi^* - \frac{|D_{X,T_C}|}{|D|} dT_C^* - \frac{|D_{X,G}|}{|D|} dG^*, \quad \text{where} \quad X^* \in \{K^*, L^*, w^*, C^*, \pi^*, W^*\}. \tag{66}$$

D.2.1 Derivatives

In order to be able to apply Cramer's rule, the equations f^1 - f^6 of the equation system are differentiated with respect to all the endogenous as well as the exogenous variables.

Differentiating f^1 with respect to K^* generates

$$f_K^1 = \frac{\partial f^1}{\partial K^*} = 0. \tag{67}$$

Differentiating f^1 with respect to L^* generates

$$f_L^1 = \frac{\partial f^1}{\partial L^*} = (1-\alpha)w^*(1-T_{LW}^*). \tag{68}$$

Differentiating f^1 with respect to w^* generates

$$f_w^1 = \frac{\partial f^1}{\partial w^*} = \big(L^* + (N-L^*)\alpha\big)(1-T_{LW}^*). \tag{69}$$

Differentiating f^1 with respect to C^* generates

$$f_C^1 = \frac{\partial f^1}{\partial C^*} = -(1+T_C^*)P^*. \tag{70}$$

Differentiating f^1 with respect to π^* generates

$$f_\pi^1 = \frac{\partial f^1}{\partial \pi^*} = 0. \tag{71}$$

Differentiating f^1 with respect to W^* generates

$$f_W^1 = \frac{\partial f^1}{\partial W^*} = \rho. \tag{72}$$

Differentiating f^1 with respect to T_{LW}^* generates

$$f_{T_{LW}}^1 = \frac{\partial f^1}{\partial T_{LW}^*} = -\big(L^* + (N-L^*)\alpha\big)w^*. \tag{73}$$

Differentiating f^1 with respect to T_π^* generates

$$f_{T_\pi}^1 = \frac{\partial f^1}{\partial T_\pi^*} = 0. \tag{74}$$

Differentiating f^1 with respect to T_C^* generates

$$f_{T_C}^1 = \frac{\partial f^1}{\partial T_C^*} = -P^*C^*. \tag{75}$$

Differentiating f^1 with respect to G^* generates

$$f_G^1 = \frac{\partial f^1}{\partial G^*} = 0. \tag{76}$$

Differentiating f^2 with respect to K^*, taking into consideration equation (41), generates

$$f_K^2 = \frac{\partial f^2}{\partial K^*} = (1 - T_\pi^*)\big(P^* F_K(K^*, L^*) - (1 + T_C^*)P^*\delta\big)$$

which, based on equation f^3 in the steady state equation system above, can be rewritten as

$$f_K^2 = (1 - T_\pi^*)P^*(1 + T_C^*)\rho. \tag{77}$$

Differentiating f^2 with respect to L^*, taking into consideration equation (48), generates

$$f_L^2 = \frac{\partial f^2}{\partial L^*} = (1 - T_\pi^*)\big(P^* F_L(K^*, L^*) - w^*\big)$$

which, based on equation f^4 in the steady state equation system above, can be rewritten as

$$f_L^2 = 0. \tag{78}$$

Differentiating f^2 with respect to w^* generates

$$f_w^2 = \frac{\partial f^2}{\partial w^*} = -(1 - T_\pi^*)L^*. \tag{79}$$

Differentiating f^2 with respect to C^* generates

$$f_C^2 = \frac{\partial f^2}{\partial C^*} = 0. \tag{80}$$

Differentiating f^2 with respect to π^* generates

$$f_\pi^2 = \frac{\partial f^2}{\partial \pi^*} = -1. \tag{81}$$

Differentiating f^2 with respect to W^* generates

$$f^2_W = \frac{\partial f^2}{\partial W^*} = 0. \tag{82}$$

Differentiating f^2 with respect to T^*_{LW} generates

$$f^2_{T_{LW}} = \frac{\partial f^2}{\partial T^*_{LW}} = 0. \tag{83}$$

Differentiating f^2 with respect to T^*_π, taking into consideration equation (12), generates

$$f^2_{T_\pi} = \frac{\partial f^2}{\partial T^*_\pi} = -P^*F(K^*, L^*) + w^*L^* + (1 + T^*_C)P^*\delta K^*. \tag{84}$$

Differentiating f^2 with respect to T^*_C generates

$$f^2_{T_C} = \frac{\partial f^2}{\partial T^*_C} = -(1 - T^*_\pi)P^*\delta K^*. \tag{85}$$

Differentiating f^2 with respect to G^* generates

$$f^2_G = \frac{\partial f^2}{\partial G^*} = 0. \tag{86}$$

Differentiating f^3 with respect to K^*, taking into consideration equation (44), generates

$$f^3_K = \frac{\partial f^3}{\partial K^*} = F_{KK}(K^*, L^*). \tag{87}$$

Differentiating f^3 with respect to L^*, taking into consideration equation (55), generates

$$f^3_L = \frac{\partial f^3}{\partial L^*} = F_{LK}(K^*, L^*). \tag{88}$$

Differentiating f^3 with respect to w^* generates

$$f_w^3 = \frac{\partial f^3}{\partial w^*} = 0. \tag{89}$$

Differentiating f^3 with respect to C^* generates

$$f_C^3 = \frac{\partial f^3}{\partial C^*} = 0. \tag{90}$$

Differentiating f^3 with respect to π^* generates

$$f_\pi^3 = \frac{\partial f^3}{\partial \pi^*} = 0. \tag{91}$$

Differentiating f^3 with respect to W^* generates

$$f_W^3 = \frac{\partial f^3}{\partial W^*} = 0. \tag{92}$$

Differentiating f^3 with respect to T_{LW}^* generates

$$f_{T_{LW}}^3 = \frac{\partial f^3}{\partial T_{LW}^*} = 0. \tag{93}$$

Differentiating f^3 with respect to T_π^* generates

$$f_{T_\pi}^3 = \frac{\partial f^3}{\partial T_\pi^*} = 0. \tag{94}$$

Differentiating f^3 with respect to T_C^* generates

$$f_{T_C}^3 = \frac{\partial f^3}{\partial T_C^*} = -(\rho + \delta). \tag{95}$$

Differentiating f^3 with respect to G^* generates

$$f_G^3 = \frac{\partial f^3}{\partial G^*} = 0. \tag{96}$$

Differentiating f^4 with respect to K^*, taking into consideration equation (55), generates

$$f_K^4 = \frac{\partial f^4}{\partial K^*} = P^* F_{LK}(K^*, L^*). \tag{97}$$

Differentiating f^4 with respect to L^*, taking into consideration (54), generates

$$f_L^4 = \frac{\partial f^4}{\partial L^*} = P^* F_{LL}(K^*, L^*). \tag{98}$$

Differentiating f^4 with respect to w^* generates

$$f_w^4 = \frac{\partial f^4}{\partial w^*} = -1. \tag{99}$$

Differentiating f^4 with respect to C^* generates

$$f_C^4 = \frac{\partial f^4}{\partial C^*} = 0. \tag{100}$$

Differentiating f^4 with respect to π^* generates

$$f_\pi^4 = \frac{\partial f^4}{\partial \pi^*} = 0. \tag{101}$$

Differentiating f^4 with respect to W^* generates

$$f_W^4 = \frac{\partial f^4}{\partial W^*} = 0. \tag{102}$$

Differentiating f^4 with respect to T_{LW}^* generates

$$f_{T_{LW}}^4 = \frac{\partial f^4}{\partial T_{LW}^*} = 0. \tag{103}$$

Differentiating f^4 with respect to T_π^* generates

$$f^4_{T_\pi} = \frac{\partial f^4}{\partial T_\pi^*} = 0. \tag{104}$$

Differentiating f^4 with respect to T_C^* generates

$$f^4_{T_C} = \frac{\partial f^4}{\partial T_C^*} = 0. \tag{105}$$

Differentiating f^4 with respect to G^* generates

$$f^4_{G} = \frac{\partial f^4}{\partial G^*} = 0. \tag{106}$$

Differentiating f^5 with respect to K^*, considering equations (54) and (58), generates

$$f^5_{K} = \frac{\partial f^5}{\partial K^*} = 0. \tag{107}$$

Differentiating f^5 with respect to L^*, considering equations (54), (58), generates

$$\begin{aligned} f^5_L = \frac{\partial f^5}{\partial L^*} = &\Big((1-\alpha)(N-L^*) - \big(L^* + (N-L^*)\alpha\big)\Big) \\ &\times (1-T^*_{LW})P^* F_{LL}(K^*, L^*) - (1-\alpha)w^*(1-T^*_{LW}). \end{aligned} \tag{108}$$

Differentiating f^5 with respect to w^* generates

$$f^5_w = \frac{\partial f^5}{\partial w^*} = (1-\alpha)(N-L^*)(1-T^*_{LW}). \tag{109}$$

Differentiating f^5 with respect to C^* generates

$$f^5_C = \frac{\partial f^5}{\partial C^*} = -\psi(1+T^*_C)P^*. \tag{110}$$

Differentiating f^5 with respect to π^* generates

$$f^5_\pi = \frac{\partial f^5}{\partial \pi^*} = 0. \tag{111}$$

Differentiating f^5 with respect to W^* generates

$$f^5_W = \frac{\partial f^5}{\partial W^*} = 0. \tag{112}$$

Differentiating f^5 with respect to T^*_{LW}, considering equations (54), generates

$$\begin{aligned} f^5_{T_{LW}} = \frac{\partial f^5}{\partial T^*_{LW}} &= -\big(L^* + (N - L^*)\alpha\big)(N - L^*)P^* F_{LL}(K^*, L^*) \\ - (1-\alpha)(N - L^*)w^*. \end{aligned} \tag{113}$$

Differentiating f^5 with respect to T^*_π generates

$$f^5_{T_\pi} = \frac{\partial f^5}{\partial T^*_\pi} = 0. \tag{114}$$

Differentiating with respect to T^*_C generates

$$f^5_{T_C} = \frac{\partial f^5}{\partial T^*_C} = -\psi P^* C^*. \tag{115}$$

Differentiating f^5 with respect to G^* generates

$$f^5_G = \frac{\partial f^5}{\partial G^*} = 0. \tag{116}$$

Differentiating f^6 with respect to K^*, taking into consideration equation (41), generates

$$f^6_K = \frac{\partial f^6}{\partial K^*} = P^* F_K(K^*, L^*) - P^*\delta. \tag{117}$$

Differentiating f^6 with respect to L^*, taking into consideration equation (48), generates

$$f^6_L = \frac{\partial f^6}{\partial L^*} = P^* F_L(K^*, L^*),$$

which, based on equation f^4 in the steady state equation system above, can be rewritten as

$$f_L^6 = w^*. \tag{118}$$

Differentiating f^6 with respect to w^* generates

$$f_w^6 = \frac{\partial f^6}{\partial w^*} = 0. \tag{119}$$

Differentiating f^6 with respect to C^* generates

$$f_C^6 = \frac{\partial f^6}{\partial C^*} = -P^*. \tag{120}$$

Differentiating f^6 with respect to π^* generates

$$f_\pi^6 = \frac{\partial f^6}{\partial \pi^*} = -1. \tag{121}$$

Differentiating f^6 with respect to W^* generates

$$f_W^6 = \frac{\partial f^6}{\partial W^*} = 0. \tag{122}$$

Differentiating f^6 with respect to T_{LW}^* generates

$$f_{T_{LW}}^6 = \frac{\partial f^6}{\partial T_{LW}^*} = 0. \tag{123}$$

Differentiating f^6 with respect to T_π^* generates

$$f_{T_\pi}^6 = \frac{\partial f^6}{\partial T_\pi^*} = 0. \tag{124}$$

Differentiating f^6 with respect to T_C^* generates

$$f_{T_C}^6 = \frac{\partial f^6}{\partial T_C^*} = 0. \tag{125}$$

Differentiating f^6 with respect to G^* generates

$$f^6_G = \frac{\partial f^6}{\partial G^*} = -P^*. \tag{126}$$

D.2.2 Determinants

Based on the theory about the application of Cramer's rule and the equations (67) - (126), the required determinants can now be calculated.

$$|D| = \begin{vmatrix} 0 & f^1_L & f^1_w & f^1_C & 0 & \rho \\ f^2_K & 0 & f^2_w & 0 & -1 & 0 \\ f^3_K & f^3_L & 0 & 0 & 0 & 0 \\ f^4_K & f^4_L & f^4_w & 0 & 0 & 0 \\ 0 & f^5_L & f^5_w & f^5_C & 0 & 0 \\ f^6_K & f^6_L & 0 & f^6_C & -1 & 0 \end{vmatrix}$$

$$= -\rho \begin{vmatrix} f^2_K & 0 & f^2_w & 0 & -1 \\ f^3_K & f^3_L & 0 & 0 & 0 \\ f^4_K & f^4_L & f^4_w & 0 & 0 \\ 0 & f^5_L & f^5_w & f^5_C & 0 \\ f^6_K & f^6_L & 0 & f^6_C & -1 \end{vmatrix} = \rho \begin{vmatrix} f^3_K & f^3_L & 0 & 0 \\ f^4_K & f^4_L & f^4_w & 0 \\ 0 & f^5_L & f^5_w & f^5_C \\ f^6_K & f^6_L & 0 & f^6_C \end{vmatrix}$$

$$+\rho \begin{vmatrix} f^2_K & 0 & f^2_w & 0 \\ f^3_K & f^3_L & 0 & 0 \\ f^4_K & f^4_L & f^4_w & 0 \\ 0 & f^5_L & f^5_w & f^5_C \end{vmatrix} = -\rho f^5_C \begin{vmatrix} f^3_K & f^3_L & 0 \\ f^4_K & f^4_L & f^4_w \\ f^6_K & f^6_L & 0 \end{vmatrix} + \rho f^6_C \begin{vmatrix} f^3_K & f^3_L & 0 \\ f^4_K & f^4_L & f^4_w \\ 0 & f^5_L & f^5_w \end{vmatrix}$$

$$+\rho f^5_C \begin{vmatrix} f^2_K & 0 & f^2_w \\ f^3_K & f^3_L & 0 \\ f^4_K & f^4_L & f^4_w \end{vmatrix}$$

$$\begin{aligned} \Leftrightarrow |D| &= \rho f^5_C f^4_w \big(f^3_K f^6_L - f^6_K f^3_L\big) \\ &+ \rho f^6_C \Big(- f^4_w f^3_K f^5_L + f^5_w (f^3_K f^4_L - f^4_K f^3_L)\Big) \\ &+ \rho f^5_C \Big(f^2_w (f^3_K f^4_L - f^4_K f^3_L) + f^4_w f^2_K f^3_L \Big). \end{aligned} \tag{127}$$

$$|D_{K,T_{LW}}| = \begin{vmatrix} f^1_{T_{LW}} & f^1_L & f^1_w & f^1_C & 0 & \rho \\ 0 & 0 & f^2_w & 0 & -1 & 0 \\ 0 & f^3_L & 0 & 0 & 0 & 0 \\ 0 & f^4_L & f^4_w & 0 & 0 & 0 \\ f^5_{T_{LW}} & f^5_L & f^5_w & f^5_C & 0 & 0 \\ 0 & f^6_L & 0 & f^6_C & -1 & 0 \end{vmatrix}$$

$$= -\rho \begin{vmatrix} 0 & 0 & f^2_w & 0 & -1 \\ 0 & f^3_L & 0 & 0 & 0 \\ 0 & f^4_L & f^4_w & 0 & 0 \\ f^5_{T_{LW}} & f^5_L & f^5_w & f^5_C & 0 \\ 0 & f^6_L & 0 & f^6_C & -1 \end{vmatrix} = \rho \begin{vmatrix} 0 & f^3_L & 0 & 0 \\ 0 & f^4_L & f^4_w & 0 \\ f^5_{T_{LW}} & f^5_L & f^5_w & f^5_C \\ 0 & f^6_L & 0 & f^6_C \end{vmatrix}$$

$$+\rho \begin{vmatrix} 0 & 0 & f^2_w & 0 \\ 0 & f^3_L & 0 & 0 \\ 0 & f^4_L & f^4_w & 0 \\ f^5_{T_{LW}} & f^5_L & f^5_w & f^5_C \end{vmatrix} = -\rho f^5_C \begin{vmatrix} 0 & f^3_L & 0 \\ 0 & f^4_L & f^4_w \\ 0 & f^6_L & 0 \end{vmatrix} + \rho f^6_C \begin{vmatrix} 0 & f^3_L & 0 \\ 0 & f^4_L & f^4_w \\ f^5_{T_{LW}} & f^5_L & f^5_w \end{vmatrix}$$

$$+\rho f^5_C \begin{vmatrix} 0 & 0 & f^2_w \\ 0 & f^3_L & 0 \\ 0 & f^4_L & f^4_w \end{vmatrix}$$

$$\Leftrightarrow |D_{K,T_{LW}}| = \rho f^6_C f^5_{T_{LW}} f^3_L f^4_w. \tag{128}$$

$$|D_{K,T_\pi}| = \begin{vmatrix} 0 & f^1_L & f^1_w & f^1_C & 0 & \rho \\ f^2_{T_\pi} & 0 & f^2_w & 0 & -1 & 0 \\ 0 & f^3_L & 0 & 0 & 0 & 0 \\ 0 & f^4_L & f^4_w & 0 & 0 & 0 \\ 0 & f^5_L & f^5_w & f^5_C & 0 & 0 \\ 0 & f^6_L & 0 & f^6_C & -1 & 0 \end{vmatrix}$$

$$= -\rho \begin{vmatrix} f^2_{T_\pi} & 0 & f^2_w & 0 & -1 \\ 0 & f^3_L & 0 & 0 & 0 \\ 0 & f^4_L & f^4_w & 0 & 0 \\ 0 & f^5_L & f^5_w & f^5_C & 0 \\ 0 & f^6_L & 0 & f^6_C & -1 \end{vmatrix} = \rho \begin{vmatrix} 0 & f^3_L & 0 & 0 \\ 0 & f^4_L & f^4_w & 0 \\ 0 & f^5_L & f^5_w & f^5_C \\ 0 & f^6_L & 0 & f^6_C \end{vmatrix}$$

$$+\rho \begin{vmatrix} f^2_{T_\pi} & 0 & f^2_w & 0 \\ 0 & f^3_L & 0 & 0 \\ 0 & f^4_L & f^4_w & 0 \\ 0 & f^5_L & f^5_w & f^5_C \end{vmatrix} = \rho f^5_C \begin{vmatrix} f^2_{T_\pi} & 0 & f^2_w \\ 0 & f^3_L & 0 \\ 0 & f^4_L & f^4_w \end{vmatrix}$$

$$\Leftrightarrow |D_{K,T_\pi}| = \rho f^5_C f^2_{T_\pi} f^3_L f^4_w. \tag{129}$$

$$|D_{K,T_C}| = \begin{vmatrix} f^1_{T_C} & f^1_L & f^1_w & f^1_C & 0 & \rho \\ f^2_{T_C} & 0 & f^2_w & 0 & -1 & 0 \\ f^3_{T_C} & f^3_L & 0 & 0 & 0 & 0 \\ 0 & f^4_L & f^4_w & 0 & 0 & 0 \\ f^5_{T_C} & f^5_L & f^5_w & f^5_C & 0 & 0 \\ 0 & f^6_L & 0 & f^6_C & -1 & 0 \end{vmatrix}$$

$$= -\rho \begin{vmatrix} f^2_{T_C} & 0 & f^2_w & 0 & -1 \\ f^3_{T_C} & f^3_L & 0 & 0 & 0 \\ 0 & f^4_L & f^4_w & 0 & 0 \\ f^5_{T_C} & f^5_L & f^5_w & f^5_C & 0 \\ 0 & f^6_L & 0 & f^6_C & -1 \end{vmatrix} = \rho \begin{vmatrix} f^3_{T_C} & f^3_L & 0 & 0 \\ 0 & f^4_L & f^4_w & 0 \\ f^5_{T_C} & f^5_L & f^5_w & f^5_C \\ 0 & f^6_L & 0 & f^6_C \end{vmatrix}$$

$$+ \rho \begin{vmatrix} f^2_{T_C} & 0 & f^2_w & 0 \\ f^3_{T_C} & f^3_L & 0 & 0 \\ 0 & f^4_L & f^4_w & 0 \\ f^5_{T_C} & f^5_L & f^5_w & f^5_C \end{vmatrix} = -\rho f^5_C \begin{vmatrix} f^3_{T_C} & f^3_L & 0 \\ 0 & f^4_L & f^4_w \\ 0 & f^6_L & 0 \end{vmatrix} + \rho f^6_C \begin{vmatrix} f^3_{T_C} & f^3_L & 0 \\ 0 & f^4_L & f^4_w \\ f^5_{T_C} & f^5_L & f^5_w \end{vmatrix}$$

$$+\rho f^5_C \begin{vmatrix} f^2_{T_C} & 0 & f^2_w \\ f^3_{T_C} & f^3_L & 0 \\ 0 & f^4_L & f^4_w \end{vmatrix}$$

$$\begin{aligned} \Leftrightarrow |D_{K,T_C}| = \rho f^5_C f^3_{T_C} f^6_L f^4_w + \rho f^6_C \big(f^3_{T_C} (f^4_L f^5_w - f^5_L f^4_w) + f^5_{T_C} f^3_L f^4_w \big) \\ + \rho f^5_C \big(f^2_{T_C} f^3_L f^4_w + f^3_{T_C} f^4_L f^2_w \big). \end{aligned} \tag{130}$$

$$|D_{K,G}| = \begin{vmatrix} 0 & f_L^1 & f_w^1 & f_C^1 & 0 & \rho \\ 0 & 0 & f_w^2 & 0 & -1 & 0 \\ 0 & f_L^3 & 0 & 0 & 0 & 0 \\ 0 & f_L^4 & f_w^4 & 0 & 0 & 0 \\ 0 & f_L^5 & f_w^5 & f_C^5 & 0 & 0 \\ f_G^6 & f_L^6 & 0 & f_C^6 & -1 & 0 \end{vmatrix}$$

$$= -\rho \begin{vmatrix} 0 & 0 & f_w^2 & 0 & -1 \\ 0 & f_L^3 & 0 & 0 & 0 \\ 0 & f_L^4 & f_w^4 & 0 & 0 \\ 0 & f_L^5 & f_w^5 & f_C^5 & 0 \\ f_G^6 & f_L^6 & 0 & f_C^6 & -1 \end{vmatrix} = \rho \begin{vmatrix} 0 & f_L^3 & 0 & 0 \\ 0 & f_L^4 & f_w^4 & 0 \\ 0 & f_L^5 & f_w^5 & f_C^5 \\ f_G^6 & f_L^6 & 0 & f_C^6 \end{vmatrix}$$

$$+\rho \begin{vmatrix} 0 & 0 & f_w^2 & 0 \\ 0 & f_L^3 & 0 & 0 \\ 0 & f_L^4 & f_w^4 & 0 \\ 0 & f_L^5 & f_w^5 & f_C^5 \end{vmatrix} = -\rho f_G^6 \begin{vmatrix} f_L^3 & 0 & 0 \\ f_L^4 & f_w^4 & 0 \\ f_L^5 & f_w^5 & f_C^5 \end{vmatrix}$$

$$\Leftrightarrow |D_{K,G}| = -\rho f_G^6 f_C^5 f_L^3 f_w^4. \tag{131}$$

$$|D_{L,T_{LW}}| = \begin{vmatrix} 0 & f_{T_{LW}}^1 & f_w^1 & f_C^1 & 0 & \rho \\ f_K^2 & 0 & f_w^2 & 0 & -1 & 0 \\ f_K^3 & 0 & 0 & 0 & 0 & 0 \\ f_K^4 & 0 & f_w^4 & 0 & 0 & 0 \\ 0 & f_{T_{LW}}^5 & f_w^5 & f_C^5 & 0 & 0 \\ f_K^6 & 0 & 0 & f_C^6 & -1 & 0 \end{vmatrix}$$

$$= -\rho \begin{vmatrix} f_K^2 & 0 & f_w^2 & 0 & -1 \\ f_K^3 & 0 & 0 & 0 & 0 \\ f_K^4 & 0 & f_w^4 & 0 & 0 \\ 0 & f_{T_{LW}}^5 & f_w^5 & f_C^5 & 0 \\ f_K^6 & 0 & 0 & f_C^6 & -1 \end{vmatrix} = \rho \begin{vmatrix} f_K^3 & 0 & 0 & 0 \\ f_K^4 & 0 & f_w^4 & 0 \\ 0 & f_{T_{LW}}^5 & f_w^5 & f_C^5 \\ f_K^6 & 0 & 0 & f_C^6 \end{vmatrix}$$

$$+\rho \begin{vmatrix} f_K^2 & 0 & f_w^2 & 0 \\ f_K^3 & 0 & 0 & 0 \\ f_K^4 & 0 & f_w^4 & 0 \\ 0 & f_{T_{LW}}^5 & f_w^5 & f_C^5 \end{vmatrix} = \rho f_K^3 \begin{vmatrix} 0 & f_w^4 & 0 \\ f_{T_{LW}}^5 & f_w^5 & f_C^5 \\ 0 & 0 & f_C^6 \end{vmatrix} + \rho f_C^5 \begin{vmatrix} f_K^2 & 0 & f_w^2 \\ f_K^3 & 0 & 0 \\ f_K^4 & 0 & f_w^4 \end{vmatrix}$$

$$\Leftrightarrow |D_{L,T_{LW}}| = -\rho f_K^3 f_C^6 f_{T_{LW}}^5 f_w^4. \tag{132}$$

$$|D_{L,T_\pi}| = \begin{vmatrix} 0 & 0 & f_w^1 & f_C^1 & 0 & \rho \\ f_K^2 & f_{T_\pi}^2 & f_w^2 & 0 & -1 & 0 \\ f_K^3 & 0 & 0 & 0 & 0 & 0 \\ f_K^4 & 0 & f_w^4 & 0 & 0 & 0 \\ 0 & 0 & f_w^5 & f_C^5 & 0 & 0 \\ f_K^6 & 0 & 0 & f_C^6 & -1 & 0 \end{vmatrix}$$

$$= -\rho \begin{vmatrix} f_K^2 & f_{T_\pi}^2 & f_w^2 & 0 & -1 \\ f_K^3 & 0 & 0 & 0 & 0 \\ f_K^4 & 0 & f_w^4 & 0 & 0 \\ 0 & 0 & f_w^5 & f_C^5 & 0 \\ f_K^6 & 0 & 0 & f_C^6 & -1 \end{vmatrix} = \rho \begin{vmatrix} f_K^3 & 0 & 0 & 0 \\ f_K^4 & 0 & f_w^4 & 0 \\ 0 & 0 & f_w^5 & f_C^5 \\ f_K^6 & 0 & 0 & f_C^6 \end{vmatrix}$$

$$+\rho \begin{vmatrix} f_K^2 & f_{T_\pi}^2 & f_w^2 & 0 \\ f_K^3 & 0 & 0 & 0 \\ f_K^4 & 0 & f_w^4 & 0 \\ 0 & 0 & f_w^5 & f_C^5 \end{vmatrix} = \rho f_C^5 \begin{vmatrix} f_K^2 & f_{T_\pi}^2 & f_w^2 \\ f_K^3 & 0 & 0 \\ f_K^4 & 0 & f_w^4 \end{vmatrix}$$

$$\Leftrightarrow |D_{L,T_\pi}| = -\rho f_C^5 f_{T_\pi}^2 f_K^3 f_w^4. \tag{133}$$

$$|D_{L,T_C}| = \begin{vmatrix} 0 & f_{T_C}^1 & f_w^1 & f_C^1 & 0 & \rho \\ f_K^2 & f_{T_C}^2 & f_w^2 & 0 & -1 & 0 \\ f_K^3 & f_{T_C}^3 & 0 & 0 & 0 & 0 \\ f_K^4 & 0 & f_w^4 & 0 & 0 & 0 \\ 0 & f_{T_C}^5 & f_w^5 & f_C^5 & 0 & 0 \\ f_K^6 & 0 & 0 & f_C^6 & -1 & 0 \end{vmatrix}$$

$$= -\rho \begin{vmatrix} f_K^2 & f_{T_C}^2 & f_w^2 & 0 & -1 \\ f_K^3 & f_{T_C}^3 & 0 & 0 & 0 \\ f_K^4 & 0 & f_w^4 & 0 & 0 \\ 0 & f_{T_C}^5 & f_w^5 & f_C^5 & 0 \\ f_K^6 & 0 & 0 & f_C^6 & -1 \end{vmatrix} = \rho \begin{vmatrix} f_K^3 & f_{T_C}^3 & 0 & 0 \\ f_K^4 & 0 & f_w^4 & 0 \\ 0 & f_{T_C}^5 & f_w^5 & f_C^5 \\ f_K^6 & 0 & 0 & f_C^6 \end{vmatrix}$$

$$
+\rho \begin{vmatrix} f_K^2 & f_{T_C}^2 & f_w^2 & 0 \\ f_K^3 & f_{T_C}^3 & 0 & 0 \\ f_K^4 & 0 & f_w^4 & 0 \\ 0 & f_{T_C}^5 & f_w^5 & f_C^5 \end{vmatrix} = -\rho f_C^5 \begin{vmatrix} f_K^3 & f_{T_C}^3 & 0 \\ f_K^4 & 0 & f_w^4 \\ f_K^6 & 0 & 0 \end{vmatrix} + \rho f_C^6 \begin{vmatrix} f_K^3 & f_{T_C}^3 & 0 \\ f_K^4 & 0 & f_w^4 \\ 0 & f_{T_C}^5 & f_w^5 \end{vmatrix}
$$

$$
+\rho f_C^5 \begin{vmatrix} f_K^2 & f_{T_C}^2 & f_w^2 \\ f_K^3 & f_{T_C}^3 & 0 \\ f_K^4 & 0 & f_w^4 \end{vmatrix}
$$

$$
\begin{aligned}
&\Leftrightarrow |D_{L,T_C}| = -\rho f_C^5 f_{T_C}^3 f_K^6 f_w^4 + \rho f_C^6 \big(- f_w^4 f_K^3 f_{T_C}^5 - f_w^5 f_K^4 f_{T_C}^3 \big) \\
&+ \rho f_C^5 \big(- f_w^2 f_K^4 f_{T_C}^3 + f_w^4 (f_K^2 f_{T_C}^3 - f_K^3 f_{T_C}^2) \big).
\end{aligned} \tag{134}
$$

$$
|D_{L,G}| = \begin{vmatrix} 0 & 0 & f_w^1 & f_C^1 & 0 & \rho \\ f_K^2 & 0 & f_w^2 & 0 & -1 & 0 \\ f_K^3 & 0 & 0 & 0 & 0 & 0 \\ f_K^4 & 0 & f_w^4 & 0 & 0 & 0 \\ 0 & 0 & f_w^5 & f_C^5 & 0 & 0 \\ f_K^6 & f_G^6 & 0 & f_C^6 & -1 & 0 \end{vmatrix}
$$

$$
= -\rho \begin{vmatrix} f_K^2 & 0 & f_w^2 & 0 & -1 \\ f_K^3 & 0 & 0 & 0 & 0 \\ f_K^4 & 0 & f_w^4 & 0 & 0 \\ 0 & 0 & f_w^5 & f_C^5 & 0 \\ f_K^6 & f_G^6 & 0 & f_C^6 & -1 \end{vmatrix} = \rho \begin{vmatrix} f_K^3 & 0 & 0 & 0 \\ f_K^4 & 0 & f_w^4 & 0 \\ 0 & 0 & f_w^5 & f_C^5 \\ f_K^6 & f_G^6 & 0 & f_C^6 \end{vmatrix}
$$

$$
+\rho \begin{vmatrix} f_K^2 & 0 & f_w^2 & 0 \\ f_K^3 & 0 & 0 & 0 \\ f_K^4 & 0 & f_w^4 & 0 \\ 0 & 0 & f_w^5 & f_C^5 \end{vmatrix} = \rho f_G^6 \begin{vmatrix} f_K^3 & 0 & 0 \\ f_K^4 & f_w^4 & 0 \\ 0 & f_w^5 & f_C^5 \end{vmatrix}
$$

$$
\Leftrightarrow |D_{L,G}| = \rho f_G^6 f_C^5 f_K^3 f_w^4. \tag{135}
$$

$$
|D_{w,T_{LW}}| = \begin{vmatrix} 0 & f_L^1 & f_{T_{LW}}^1 & f_C^1 & 0 & \rho \\ f_K^2 & 0 & 0 & 0 & -1 & 0 \\ f_K^3 & f_L^3 & 0 & 0 & 0 & 0 \\ f_K^4 & f_L^4 & 0 & 0 & 0 & 0 \\ 0 & f_L^5 & f_{T_{LW}}^5 & f_C^5 & 0 & 0 \\ f_K^6 & f_L^6 & 0 & f_C^6 & -1 & 0 \end{vmatrix}
$$

$$= -\rho \begin{vmatrix} f_K^2 & 0 & 0 & 0 & -1 \\ f_K^3 & f_L^3 & 0 & 0 & 0 \\ f_K^4 & f_L^4 & 0 & 0 & 0 \\ 0 & f_L^5 & f_{T_{LW}}^5 & f_C^5 & 0 \\ f_K^6 & f_L^6 & 0 & f_C^6 & -1 \end{vmatrix} = \rho \begin{vmatrix} f_K^3 & f_L^3 & 0 & 0 \\ f_K^4 & f_L^4 & 0 & 0 \\ 0 & f_L^5 & f_{T_{LW}}^5 & f_C^5 \\ f_K^6 & f_L^6 & 0 & f_C^6 \end{vmatrix}$$

$$+\rho \begin{vmatrix} f_K^2 & 0 & 0 & 0 \\ f_K^3 & f_L^3 & 0 & 0 \\ f_K^4 & f_L^4 & 0 & 0 \\ 0 & f_L^5 & f_{T_{LW}}^5 & f_C^5 \end{vmatrix} = -\rho f_C^5 \begin{vmatrix} f_K^3 & f_L^3 & 0 \\ f_K^4 & f_L^4 & 0 \\ f_K^6 & f_L^6 & 0 \end{vmatrix} + \rho f_C^6 \begin{vmatrix} f_K^3 & f_L^3 & 0 \\ f_K^4 & f_L^4 & 0 \\ 0 & f_L^5 & f_{T_{LW}}^5 \end{vmatrix}$$

$$+\rho f_C^5 \begin{vmatrix} f_K^2 & 0 & 0 \\ f_K^3 & f_L^3 & 0 \\ f_K^4 & f_L^4 & 0 \end{vmatrix}$$

$$\Leftrightarrow |D_{w,T_{LW}}| = \rho f_C^6 f_{T_{LW}}^5 (f_K^3 f_L^4 - f_K^4 f_L^3). \tag{136}$$

$$|D_{w,T_\pi}| = \begin{vmatrix} 0 & f_L^1 & 0 & f_C^1 & 0 & \rho \\ f_K^2 & 0 & f_{T_\pi}^2 & 0 & -1 & 0 \\ f_K^3 & f_L^3 & 0 & 0 & 0 & 0 \\ f_K^4 & f_L^4 & 0 & 0 & 0 & 0 \\ 0 & f_L^5 & 0 & f_C^5 & 0 & 0 \\ f_K^6 & f_L^6 & 0 & f_C^6 & -1 & 0 \end{vmatrix}$$

$$= -\rho \begin{vmatrix} f_K^2 & 0 & f_{T_\pi}^2 & 0 & -1 \\ f_K^3 & f_L^3 & 0 & 0 & 0 \\ f_K^4 & f_L^4 & 0 & 0 & 0 \\ 0 & f_L^5 & 0 & f_C^5 & 0 \\ f_K^6 & f_L^6 & 0 & f_C^6 & -1 \end{vmatrix} = \rho \begin{vmatrix} f_K^3 & f_L^3 & 0 & 0 \\ f_K^4 & f_L^4 & 0 & 0 \\ 0 & f_L^5 & 0 & f_C^5 \\ f_K^6 & f_L^6 & 0 & f_C^6 \end{vmatrix}$$

$$+\rho \begin{vmatrix} f_K^2 & 0 & f_{T_\pi}^2 & 0 \\ f_K^3 & f_L^3 & 0 & 0 \\ f_K^4 & f_L^4 & 0 & 0 \\ 0 & f_L^5 & 0 & f_C^5 \end{vmatrix} = \rho f_C^5 \begin{vmatrix} f_K^2 & 0 & f_{T_\pi}^2 \\ f_K^3 & f_L^3 & 0 \\ f_K^4 & f_L^4 & 0 \end{vmatrix}$$

$$\Leftrightarrow |D_{w,T_\pi}| = \rho f_C^5 f_{T_\pi}^2 (f_K^3 f_L^4 - f_K^4 f_L^3). \tag{137}$$

$$|D_{w,T_C}| = \begin{vmatrix} 0 & f_L^1 & f_{T_C}^1 & f_C^1 & 0 & \rho \\ f_K^2 & 0 & f_{T_C}^2 & 0 & -1 & 0 \\ f_K^3 & f_L^3 & f_{T_C}^3 & 0 & 0 & 0 \\ f_K^4 & f_L^4 & 0 & 0 & 0 & 0 \\ 0 & f_L^5 & f_{T_C}^5 & f_C^5 & 0 & 0 \\ f_K^6 & f_L^6 & 0 & f_C^6 & -1 & 0 \end{vmatrix}$$

$$= -\rho \begin{vmatrix} f_K^2 & 0 & f_{T_C}^2 & 0 & -1 \\ f_K^3 & f_L^3 & f_{T_C}^3 & 0 & 0 \\ f_K^4 & f_L^4 & 0 & 0 & 0 \\ 0 & f_L^5 & f_{T_C}^5 & f_C^5 & 0 \\ f_K^6 & f_L^6 & 0 & f_C^6 & -1 \end{vmatrix} = \rho \begin{vmatrix} f_K^3 & f_L^3 & f_{T_C}^3 & 0 \\ f_K^4 & f_L^4 & 0 & 0 \\ 0 & f_L^5 & f_{T_C}^5 & f_C^5 \\ f_K^6 & f_L^6 & 0 & f_C^6 \end{vmatrix}$$

$$+\rho \begin{vmatrix} f_K^2 & 0 & f_{T_C}^2 & 0 \\ f_K^3 & f_L^3 & f_{T_C}^3 & 0 \\ f_K^4 & f_L^4 & 0 & 0 \\ 0 & f_L^5 & f_{T_C}^5 & f_C^5 \end{vmatrix} = -\rho f_C^5 \begin{vmatrix} f_K^3 & f_L^3 & f_{T_C}^3 \\ f_K^4 & f_L^4 & 0 \\ f_K^6 & f_L^6 & 0 \end{vmatrix} + \rho f_C^6 \begin{vmatrix} f_K^3 & f_L^3 & f_{T_C}^3 \\ f_K^4 & f_L^4 & 0 \\ 0 & f_L^5 & f_{T_C}^5 \end{vmatrix}$$

$$+\rho f_C^5 \begin{vmatrix} f_K^2 & 0 & f_{T_C}^2 \\ f_K^3 & f_L^3 & f_{T_C}^3 \\ f_K^4 & f_L^4 & 0 \end{vmatrix}$$

$$\begin{aligned} &\Leftrightarrow |D_{w,T_C}| = -\rho f_C^5 f_{T_C}^3 (f_K^4 f_L^6 - f_K^6 f_L^4) \\ &+ \rho f_C^6 \big(f_{T_C}^3 f_K^4 f_L^5 + f_{T_C}^5 (f_K^3 f_L^4 - f_K^4 f_L^3) \big) \\ &+ \rho f_C^5 \big(f_{T_C}^2 (f_K^3 f_L^4 - f_K^4 f_L^3) - f_{T_C}^3 f_K^2 f_L^4 \big). \end{aligned} \tag{138}$$

$$|D_{w,G}| = \begin{vmatrix} 0 & f_L^1 & 0 & f_C^1 & 0 & \rho \\ f_K^2 & 0 & 0 & 0 & -1 & 0 \\ f_K^3 & f_L^3 & 0 & 0 & 0 & 0 \\ f_K^4 & f_L^4 & 0 & 0 & 0 & 0 \\ 0 & f_L^5 & 0 & f_C^5 & 0 & 0 \\ f_K^6 & f_L^6 & f_G^6 & f_C^6 & -1 & 0 \end{vmatrix}$$

$$= -\rho \begin{vmatrix} f_K^2 & 0 & 0 & 0 & -1 \\ f_K^3 & f_L^3 & 0 & 0 & 0 \\ f_K^4 & f_L^4 & 0 & 0 & 0 \\ 0 & f_L^5 & 0 & f_C^5 & 0 \\ f_K^6 & f_L^6 & f_G^6 & f_C^6 & -1 \end{vmatrix} = \rho \begin{vmatrix} f_K^3 & f_L^3 & 0 & 0 \\ f_K^4 & f_L^4 & 0 & 0 \\ 0 & f_L^5 & 0 & f_C^5 \\ f_K^6 & f_L^6 & f_G^6 & f_C^6 \end{vmatrix}$$

$$+\rho \begin{vmatrix} f_K^2 & 0 & 0 & 0 \\ f_K^3 & f_L^3 & 0 & 0 \\ f_K^4 & f_L^4 & 0 & 0 \\ 0 & f_L^5 & 0 & f_C^5 \end{vmatrix} = -\rho f_C^5 \begin{vmatrix} f_K^3 & f_L^3 & 0 \\ f_K^4 & f_L^4 & 0 \\ f_K^6 & f_L^6 & f_G^6 \end{vmatrix} + \rho f_C^6 \begin{vmatrix} f_K^3 & f_L^3 & 0 \\ f_K^4 & f_L^4 & 0 \\ 0 & f_L^5 & 0 \end{vmatrix}$$

$$\Leftrightarrow |D_{w,G}| = -\rho f_C^5 f_G^6 (f_K^3 f_L^4 - f_K^4 f_L^3). \tag{139}$$

$$|D_{C,T_{LW}}| = \begin{vmatrix} 0 & f_L^1 & f_w^1 & f_{T_{LW}}^1 & 0 & \rho \\ f_K^2 & 0 & f_w^2 & 0 & -1 & 0 \\ f_K^3 & f_L^3 & 0 & 0 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 & 0 & 0 \\ 0 & f_L^5 & f_w^5 & f_{T_{LW}}^5 & 0 & 0 \\ f_K^6 & f_L^6 & 0 & 0 & -1 & 0 \end{vmatrix}$$

$$= -\rho \begin{vmatrix} f_K^2 & 0 & f_w^2 & 0 & -1 \\ f_K^3 & f_L^3 & 0 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 & 0 \\ 0 & f_L^5 & f_w^5 & f_{T_{LW}}^5 & 0 \\ f_K^6 & f_L^6 & 0 & 0 & -1 \end{vmatrix} = \rho \begin{vmatrix} f_K^3 & f_L^3 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 \\ 0 & f_L^5 & f_w^5 & f_{T_{LW}}^5 \\ f_K^6 & f_L^6 & 0 & 0 \end{vmatrix}$$

$$+\rho \begin{vmatrix} f_K^2 & 0 & f_w^2 & 0 \\ f_K^3 & f_L^3 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 \\ 0 & f_L^5 & f_w^5 & f_{T_{LW}}^5 \end{vmatrix} = -\rho f_{T_{LW}}^5 \begin{vmatrix} f_K^3 & f_L^3 & 0 \\ f_K^4 & f_L^4 & f_w^4 \\ f_K^6 & f_L^6 & 0 \end{vmatrix} + \rho f_{T_{LE}}^5 \begin{vmatrix} f_K^2 & 0 & f_w^2 \\ f_K^3 & f_L^3 & 0 \\ f_K^4 & f_L^4 & f_w^4 \end{vmatrix}$$

$$\begin{aligned} \Leftrightarrow |D_{C,T_{LW}}| &= \rho f_{T_{LW}}^5 f_w^4 \left(f_K^3 f_L^6 - f_K^6 f_L^3 \right) \\ &+ \rho f_{T_{LW}}^5 f_w^2 \left(f_K^3 f_L^4 - f_K^4 f_L^3 \right) + \rho f_{T_{LW}}^5 f_w^4 f_K^2 f_L^3. \end{aligned} \tag{140}$$

$$|D_{C,T_\pi}| = \begin{vmatrix} 0 & f_L^1 & f_w^1 & 0 & 0 & \rho \\ f_K^2 & 0 & f_w^2 & f_{T_\pi}^2 & -1 & 0 \\ f_K^3 & f_L^3 & 0 & 0 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 & 0 & 0 \\ 0 & f_L^5 & f_w^5 & 0 & 0 & 0 \\ f_K^6 & f_L^6 & 0 & 0 & -1 & 0 \end{vmatrix}$$

$$= -\rho \begin{vmatrix} f_K^2 & 0 & f_w^2 & f_{T_\pi}^2 & -1 \\ f_K^3 & f_L^3 & 0 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 & 0 \\ 0 & f_L^5 & f_w^5 & 0 & 0 \\ f_K^6 & f_L^6 & 0 & 0 & -1 \end{vmatrix} = \rho \begin{vmatrix} f_K^3 & f_L^3 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 \\ 0 & f_L^5 & f_w^5 & 0 \\ f_K^6 & f_L^6 & 0 & 0 \end{vmatrix}$$

$$+\rho \begin{vmatrix} f_K^2 & 0 & f_w^2 & f_{T_\pi}^2 \\ f_K^3 & f_L^3 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 \\ 0 & f_L^5 & f_w^5 & 0 \end{vmatrix} = -\rho f_{T_\pi}^2 \begin{vmatrix} f_K^3 & f_L^3 & 0 \\ f_K^4 & f_L^4 & f_w^4 \\ 0 & f_L^5 & f_w^5 \end{vmatrix}$$

$$\Leftrightarrow |D_{C,T_\pi}| = -\rho f_{T_\pi}^2 \big(- f_w^4 f_K^3 f_L^5 + f_w^5 (f_K^3 f_L^4 - f_K^4 f_L^3) \big). \tag{141}$$

$$|D_{C,T_C}| = \begin{vmatrix} 0 & f_L^1 & f_w^1 & f_{T_C}^1 & 0 & \rho \\ f_K^2 & 0 & f_w^2 & f_{T_C}^2 & -1 & 0 \\ f_K^3 & f_L^3 & 0 & f_{T_C}^3 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 & 0 & 0 \\ 0 & f_L^5 & f_w^5 & f_{T_C}^5 & 0 & 0 \\ f_K^6 & f_L^6 & 0 & 0 & -1 & 0 \end{vmatrix}$$

$$= -\rho \begin{vmatrix} f_K^2 & 0 & f_w^2 & f_{T_C}^2 & -1 \\ f_K^3 & f_L^3 & 0 & f_{T_C}^3 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 & 0 \\ 0 & f_L^5 & f_w^5 & f_{T_C}^5 & 0 \\ f_K^6 & f_L^6 & 0 & 0 & -1 \end{vmatrix} = \rho \begin{vmatrix} f_K^3 & f_L^3 & 0 & f_{T_C}^3 \\ f_K^4 & f_L^4 & f_w^4 & 0 \\ 0 & f_L^5 & f_w^5 & f_{T_C}^5 \\ f_K^6 & f_L^6 & 0 & 0 \end{vmatrix}$$

$$+ \rho \begin{vmatrix} f_K^2 & 0 & f_w^2 & f_{T_C}^2 \\ f_K^3 & f_L^3 & 0 & f_{T_C}^3 \\ f_K^4 & f_L^4 & f_w^4 & 0 \\ 0 & f_L^5 & f_w^5 & f_{T_C}^5 \end{vmatrix} = -\rho f_{T_C}^3 \begin{vmatrix} f_K^4 & f_L^4 & f_w^4 \\ 0 & f_L^5 & f_w^5 \\ f_K^6 & f_L^6 & 0 \end{vmatrix} -\rho f_{T_C}^5 \begin{vmatrix} f_K^3 & f_L^3 & 0 \\ f_K^4 & f_L^4 & f_w^4 \\ f_K^6 & f_L^6 & 0 \end{vmatrix}$$

$$-\rho f^2_{T_C}\begin{vmatrix} f^3_K & f^3_L & 0 \\ f^4_K & f^4_L & f^4_w \\ 0 & f^5_L & f^5_w \end{vmatrix} + \rho f^3_{T_C}\begin{vmatrix} f^2_K & 0 & f^2_w \\ f^4_K & f^4_L & f^4_w \\ 0 & f^5_L & f^5_w \end{vmatrix} + \rho f^5_{T_C}\begin{vmatrix} f^2_K & 0 & f^2_w \\ f^3_K & f^3_L & 0 \\ f^4_K & f^4_L & f^4_w \end{vmatrix}$$

$$= -\rho f^3_{T_C}\Big(- f^4_w f^6_K f^5_L - f^5_w\big(f^4_K f^6_L - f^6_K f^4_L\big)\Big) + \rho f^5_{T_C} f^4_w\big(f^3_K f^6_L - f^6_K f^3_L\big)$$

$$- \rho f^2_{T_C}\Big(- f^4_w f^3_K f^5_L + f^5_w\big(f^3_K f^4_L - f^4_K f^3_L\big)\Big)$$

$$+ \rho f^3_{T_C}\Big(f^2_K\big(f^4_L f^5_w - f^5_L f^4_w\big) + f^2_w f^4_K f^5_L\Big)$$

$$+ \rho f^5_{T_C}\Big(f^2_w\big(f^3_K f^4_L - f^4_K f^3_L\big) + f^4_w f^2_K f^3_L\Big)$$

$$= -\rho f^3_{T_C}\Big(f^6_K\big(f^4_L f^5_w - f^4_w f^5_L\big) - f^5_w f^4_K f^6_L\Big)$$

$$- \rho f^2_{T_C}\Big(- f^4_w f^3_K f^5_L + f^5_w(f^3_K f^4_L - f^4_K f^3_L)\Big)$$

$$+ \rho f^3_{T_C}\Big(f^2_K\big(f^4_L f^5_w - f^5_L f^4_w\big) + f^2_w f^4_K f^5_L\Big)$$

$$+ \rho f^5_{T_C} f^2_w\big(f^3_K f^4_L - f^4_K f^3_L\big) + \rho f^5_{T_C} f^4_w\Big(\big(f^3_K f^6_L - f^6_K f^3_L\big) + f^2_K f^3_L\Big)$$

$$\Leftrightarrow |D_{C,T_C}| = -\rho f^3_{T_C}\Big[\big(f^6_K - f^2_K\big)\big(f^4_L f^5_w - f^5_L f^4_w\big) - f^4_K\big(f^6_L f^5_w + f^2_w f^5_L\big)\Big]$$

$$- \rho f^2_{T_C}\Big(- f^4_w f^3_K f^5_L + f^5_w(f^3_K f^4_L - f^4_K f^3_L)\Big) \tag{142}$$

$$+ \rho f^5_{T_C} f^2_w\big(f^3_K f^4_L - f^4_K f^3_L\big) + \rho f^5_{T_C} f^4_w\Big(\big(f^3_K f^6_L - f^6_K f^3_L\big) + f^2_K f^3_L\Big).$$

$$|D_{C,G}| = \begin{vmatrix} 0 & f^1_L & f^1_w & 0 & 0 & \rho \\ f^2_K & 0 & f^2_w & 0 & -1 & 0 \\ f^3_K & f^3_L & 0 & 0 & 0 & 0 \\ f^4_K & f^4_L & f^4_w & 0 & 0 & 0 \\ 0 & f^5_L & f^5_w & 0 & 0 & 0 \\ f^6_K & f^6_L & 0 & f^6_G & -1 & 0 \end{vmatrix}$$

$$= -\rho\begin{vmatrix} f^2_K & 0 & f^2_w & 0 & -1 \\ f^3_K & f^3_L & 0 & 0 & 0 \\ f^4_K & f^4_L & f^4_w & 0 & 0 \\ 0 & f^5_L & f^5_w & 0 & 0 \\ f^6_K & f^6_L & 0 & f^6_G & -1 \end{vmatrix} = \rho\begin{vmatrix} f^3_K & f^3_L & 0 & 0 \\ f^4_K & f^4_L & f^4_w & 0 \\ 0 & f^5_L & f^5_w & 0 \\ f^6_K & f^6_L & 0 & f^6_G \end{vmatrix}$$

$$+\rho\begin{vmatrix} f_K^2 & 0 & f_w^2 & 0 \\ f_K^3 & f_L^3 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 \\ 0 & f_L^5 & f_w^5 & 0 \end{vmatrix} = \rho f_G^6 \begin{vmatrix} f_K^3 & f_L^3 & 0 \\ f_K^4 & f_L^4 & f_w^4 \\ 0 & f_L^5 & f_w^5 \end{vmatrix}$$

$$\Leftrightarrow |D_{C,G}| = \rho f_G^6\big(-f_w^4 f_K^3 f_L^5 + f_w^5(f_K^3 f_L^4 - f_L^3 f_K^4)\big). \tag{143}$$

$$|D_{\pi,T_{LW}}| = \begin{vmatrix} 0 & f_L^1 & f_w^1 & f_C^1 & f_{T_{LW}}^1 & \rho \\ f_K^2 & 0 & f_w^2 & 0 & 0 & 0 \\ f_K^3 & f_L^3 & 0 & 0 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 & 0 & 0 \\ 0 & f_L^5 & f_w^5 & f_C^5 & f_{T_{LW}}^5 & 0 \\ f_K^6 & f_L^6 & 0 & f_C^6 & 0 & 0 \end{vmatrix}$$

$$= -\rho\begin{vmatrix} f_K^2 & 0 & f_w^2 & 0 & 0 \\ f_K^3 & f_L^3 & 0 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 & 0 \\ 0 & f_L^5 & f_w^5 & f_C^5 & f_{T_{LW}}^5 \\ f_K^6 & f_L^6 & 0 & f_C^6 & 0 \end{vmatrix} = -\rho f_C^5 \begin{vmatrix} f_K^2 & 0 & f_w^2 & 0 \\ f_K^3 & f_L^3 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 \\ f_K^6 & f_L^6 & 0 & 0 \end{vmatrix}$$

$$+\rho f_C^6\begin{vmatrix} f_K^2 & 0 & f_w^2 & 0 \\ f_K^3 & f_L^3 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 \\ 0 & f_L^5 & f_w^5 & f_{T_{LW}}^5 \end{vmatrix} = \rho f_C^6 f_{T_{LW}}^5 \begin{vmatrix} f_K^2 & 0 & f_w^2 \\ f_K^3 & f_L^3 & 0 \\ f_K^4 & f_L^4 & f_w^4 \end{vmatrix}$$

$$\Leftrightarrow |D_{\pi,T_{LW}}| = \rho f_C^6 f_{T_{LW}}^5 \left(f_w^2(f_K^3 f_L^4 - f_K^4 f_L^3) + f_w^4 f_K^2 f_L^3 \right). \tag{144}$$

$$|D_{\pi,T_\pi}| = \begin{vmatrix} 0 & f_L^1 & f_w^1 & f_C^1 & 0 & \rho \\ f_K^2 & 0 & f_w^2 & 0 & f_{T_\pi}^2 & 0 \\ f_K^3 & f_L^3 & 0 & 0 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 & 0 & 0 \\ 0 & f_L^5 & f_w^5 & f_C^5 & 0 & 0 \\ f_K^6 & f_L^6 & 0 & f_C^6 & 0 & 0 \end{vmatrix}$$

$$= -\rho \begin{vmatrix} f_K^2 & 0 & f_w^2 & 0 & f_{T_\pi}^2 \\ f_K^3 & f_L^3 & 0 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 & 0 \\ 0 & f_L^5 & f_w^5 & f_C^5 & 0 \\ f_K^6 & f_L^6 & 0 & f_C^6 & 0 \end{vmatrix} = -\rho f_{T_\pi}^2 \begin{vmatrix} f_K^3 & f_L^3 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 \\ 0 & f_L^5 & f_w^5 & f_C^5 \\ f_K^6 & f_L^6 & 0 & f_C^6 \end{vmatrix}$$

$$= \rho f_{T_\pi}^2 f_C^5 \begin{vmatrix} f_K^3 & f_L^3 & 0 \\ f_K^4 & f_L^4 & f_w^4 \\ f_K^6 & f_L^6 & 0 \end{vmatrix} - \rho f_{T_\pi}^2 f_C^6 \begin{vmatrix} f_K^3 & f_L^3 & 0 \\ f_K^4 & f_L^4 & f_w^4 \\ 0 & f_L^5 & f_w^5 \end{vmatrix}$$

$$\begin{aligned} \Leftrightarrow |D_{\pi,T_\pi}| &= -\rho f_{T_\pi}^2 f_C^5 f_w^4 \big(f_K^3 f_L^6 - f_K^6 f_L^3\big) \\ &- \rho f_{T_\pi}^2 f_C^6 \Big(- f_w^4 f_K^3 f_L^5 + f_w^5 (f_K^3 f_L^4 - f_K^4 f_L^3) \Big). \end{aligned} \tag{145}$$

$$|D_{\pi,T_C}| = \begin{vmatrix} 0 & f_L^1 & f_w^1 & f_C^1 & f_{T_C}^1 & \rho \\ f_K^2 & 0 & f_w^2 & 0 & f_{T_C}^2 & 0 \\ f_K^3 & f_L^3 & 0 & 0 & f_{T_C}^3 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 & 0 & 0 \\ 0 & f_L^5 & f_w^5 & f_C^5 & f_{T_C}^5 & 0 \\ f_K^6 & f_L^6 & 0 & f_C^6 & 0 & 0 \end{vmatrix}$$

$$= -\rho \begin{vmatrix} f_K^2 & 0 & f_w^2 & 0 & f_{T_C}^2 \\ f_K^3 & f_L^3 & 0 & 0 & f_{T_C}^3 \\ f_K^4 & f_L^4 & f_w^4 & 0 & 0 \\ 0 & f_L^5 & f_w^5 & f_C^5 & f_{T_C}^5 \\ f_K^6 & f_L^6 & 0 & f_C^6 & 0 \end{vmatrix} = -\rho f_C^5 \begin{vmatrix} f_K^2 & 0 & f_w^2 & f_{T_C}^2 \\ f_K^3 & f_L^3 & 0 & f_{T_C}^3 \\ f_K^4 & f_L^4 & f_w^4 & 0 \\ f_K^6 & f_L^6 & 0 & 0 \end{vmatrix}$$

$$+\rho f_C^6 \begin{vmatrix} f_K^2 & 0 & f_w^2 & f_{T_C}^2 \\ f_K^3 & f_L^3 & 0 & f_{T_C}^3 \\ f_K^4 & f_L^4 & f_w^4 & 0 \\ 0 & f_L^5 & f_w^5 & f_{T_C}^5 \end{vmatrix} = \rho f_C^5 f_{T_C}^2 \begin{vmatrix} f_K^3 & f_L^3 & 0 \\ f_K^4 & f_L^4 & f_w^4 \\ f_K^6 & f_L^6 & 0 \end{vmatrix} - \rho f_C^5 f_{T_C}^3 \begin{vmatrix} f_K^2 & 0 & f_w^2 \\ f_K^4 & f_L^4 & f_w^4 \\ f_K^6 & f_L^6 & 0 \end{vmatrix}$$

$$-\rho f_C^6 f_{T_C}^2 \begin{vmatrix} f_K^3 & f_L^3 & 0 \\ f_K^4 & f_L^4 & f_w^4 \\ 0 & f_L^5 & f_w^5 \end{vmatrix} + \rho f_C^6 f_{T_C}^3 \begin{vmatrix} f_K^2 & 0 & f_w^2 \\ f_K^4 & f_L^4 & f_w^4 \\ 0 & f_L^5 & f_w^5 \end{vmatrix} + \rho f_C^6 f_{T_C}^5 \begin{vmatrix} f_K^2 & 0 & f_w^2 \\ f_K^3 & f_L^3 & 0 \\ f_K^4 & f_L^4 & f_w^4 \end{vmatrix}$$

$$\begin{aligned}\Leftrightarrow |D_{\pi,T_C}| &= -\rho f_C^5 f_{T_C}^2 f_w^4\big(f_K^3 f_L^6 - f_K^6 f_L^3\big)\\ &- \rho f_C^5 f_{T_C}^3\Bigg(f_w^2\big(f_K^4 f_L^6 - f_K^6 f_L^4\big) - f_w^4 f_K^2 f_L^6\Bigg)\\ &- \rho f_C^6 f_{T_C}^2\Bigg(-f_w^4 f_K^3 f_L^5 + f_w^5\big(f_K^3 f_L^4 - f_K^4 f_L^3\big)\Bigg)\\ &+ \rho f_C^6 f_{T_C}^3\Bigg(f_K^2\big(f_L^4 f_w^5 - f_L^5 f_w^4\big) + f_w^2 f_K^4 f_L^5\Bigg)\\ &+ \rho f_C^6 f_{T_C}^5\Bigg(f_w^2\big(f_K^3 f_L^4 - f_K^4 f_L^3\big) + f_w^4 f_K^2 f_L^3\Bigg).\end{aligned} \tag{146}$$

$$|D_{\pi,G}| = \begin{vmatrix} 0 & f_L^1 & f_w^1 & f_C^1 & 0 & \rho \\ f_K^2 & 0 & f_w^2 & 0 & 0 & 0 \\ f_K^3 & f_L^3 & 0 & 0 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 & 0 & 0 \\ 0 & f_L^5 & f_w^5 & f_C^5 & 0 & 0 \\ f_K^6 & f_L^6 & 0 & f_C^6 & f_G^6 & 0 \end{vmatrix}$$

$$= -\rho \begin{vmatrix} f_K^2 & 0 & f_w^2 & 0 & 0 \\ f_K^3 & f_L^3 & 0 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 & 0 \\ 0 & f_L^5 & f_w^5 & f_C^5 & 0 \\ f_K^6 & f_L^6 & 0 & f_C^6 & f_G^6 \end{vmatrix} = \rho f_G^6 \begin{vmatrix} f_K^2 & 0 & f_w^2 & 0 \\ f_K^3 & f_L^3 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 \\ 0 & f_L^5 & f_w^5 & f_C^5 \end{vmatrix}$$

$$= \rho f_G^6 f_C^5 \begin{vmatrix} f_K^2 & 0 & f_w^2 \\ f_K^3 & f_L^3 & 0 \\ f_K^4 & f_L^4 & f_w^4 \end{vmatrix}$$

$$\Leftrightarrow |D_{\pi,G}| = \rho f_G^6 f_C^5 f_K^2 f_L^3 f_w^4 + \rho f_G^6 f_C^5 f_w^2\big(f_K^3 f_L^4 - f_K^4 f_L^3\big). \tag{147}$$

$$|D_{W,T_{LW}}| = \begin{vmatrix} 0 & f_L^1 & f_w^1 & f_C^1 & 0 & f_{T_{LW}}^1 \\ f_K^2 & 0 & f_w^2 & 0 & -1 & 0 \\ f_K^3 & f_L^3 & 0 & 0 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 & 0 & 0 \\ 0 & f_L^5 & f_w^5 & f_C^5 & 0 & f_{T_{LW}}^5 \\ f_K^6 & f_L^6 & 0 & f_C^6 & -1 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & f_L^1 & f_w^1 & f_C^1 & f_{T_{LW}}^1 \\ f_K^3 & f_L^3 & 0 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 & 0 \\ 0 & f_L^5 & f_w^5 & f_C^5 & f_{T_{LW}}^5 \\ f_K^6 & f_L^6 & 0 & f_C^6 & 0 \end{vmatrix} + \begin{vmatrix} 0 & f_L^1 & f_w^1 & f_C^1 & f_{T_{LW}}^1 \\ f_K^2 & 0 & f_w^2 & 0 & 0 \\ f_K^3 & f_L^3 & 0 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 & 0 \\ 0 & f_L^5 & f_w^5 & f_C^5 & f_{T_{LW}}^5 \end{vmatrix}$$

$$= -f_C^1 \begin{vmatrix} f_K^3 & f_L^3 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 \\ 0 & f_L^5 & f_w^5 & f_{T_{LW}}^5 \\ f_K^6 & f_L^6 & 0 & 0 \end{vmatrix} + f_C^5 \begin{vmatrix} 0 & f_L^1 & f_w^1 & f_{T_{LW}}^1 \\ f_K^3 & f_L^3 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 \\ f_K^6 & f_L^6 & 0 & 0 \end{vmatrix}$$

$$-f_C^6 \begin{vmatrix} 0 & f_L^1 & f_w^1 & f_{T_{LW}}^1 \\ f_K^3 & f_L^3 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 \\ 0 & f_L^5 & f_w^5 & f_{T_{LW}}^5 \end{vmatrix} - f_C^1 \begin{vmatrix} f_K^2 & 0 & f_w^2 & 0 \\ f_K^3 & f_L^3 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 \\ 0 & f_L^5 & f_w^5 & f_{T_{LW}}^5 \end{vmatrix}$$

$$-f_C^5 \begin{vmatrix} 0 & f_L^1 & f_w^1 & f_{T_{LW}}^1 \\ f_K^2 & 0 & f_w^2 & 0 \\ f_K^3 & f_L^3 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 \end{vmatrix} = f_C^1 f_{T_{LW}}^5 \begin{vmatrix} f_K^3 & f_L^3 & 0 \\ f_K^4 & f_L^4 & f_w^4 \\ f_K^6 & f_L^6 & 0 \end{vmatrix}$$

$$-f_C^5 f_{T_{LW}}^1 \begin{vmatrix} f_K^3 & f_L^3 & 0 \\ f_K^4 & f_L^4 & f_w^4 \\ f_K^6 & f_L^6 & 0 \end{vmatrix} + f_C^6 f_{T_{LW}}^1 \begin{vmatrix} f_K^3 & f_L^3 & 0 \\ f_K^4 & f_L^4 & f_w^4 \\ 0 & f_L^5 & f_w^5 \end{vmatrix} - f_C^6 f_{T_{LW}}^5 \begin{vmatrix} 0 & f_L^1 & f_w^1 \\ f_K^3 & f_L^3 & 0 \\ f_K^4 & f_L^4 & f_w^4 \end{vmatrix}$$

$$-f_C^1 f_{T_{LW}}^5 \begin{vmatrix} f_K^2 & 0 & f_w^2 \\ f_K^3 & f_L^3 & 0 \\ f_K^4 & f_L^4 & f_w^4 \end{vmatrix} + f_C^5 f_{T_{LW}}^1 \begin{vmatrix} f_K^2 & 0 & f_w^2 \\ f_K^3 & f_L^3 & 0 \\ f_K^4 & f_L^4 & f_w^4 \end{vmatrix}$$

$$\begin{aligned}
&= -f_C^1 f_{T_{LW}}^5 f_w^4 \big(f_K^3 f_L^6 - f_K^6 f_L^3\big) + f_C^5 f_{T_{LW}}^1 f_w^4 \big(f_K^3 f_L^6 - f_K^6 f_L^3\big) \\
&+ f_C^6 f_{T_{LW}}^1 \Bigg(- f_w^4 f_K^3 f_L^5 + f_w^5 \big(f_K^3 f_L^4 - f_K^4 f_L^3\big)\Bigg) \\
&- f_C^6 f_{T_{LW}}^5 \Bigg(f_w^1 \big(f_K^3 f_L^4 - f_K^4 f_L^3\big) - f_w^4 f_K^3 f_L^1 \Bigg) \\
&- f_C^1 f_{T_{LW}}^5 \Bigg(f_w^2 \big(f_K^3 f_L^4 - f_K^4 f_L^3\big) + f_w^4 f_K^2 f_L^3 \Bigg) \\
&+ f_C^5 f_{T_{LW}}^1 \Bigg(f_w^2 \big(f_K^3 f_L^4 - f_K^4 f_L^3\big) + f_w^4 f_K^2 f_L^3 \Bigg)
\end{aligned}$$

$$
\begin{aligned}
&\Leftrightarrow |D_{W,T_{LW}}| = -\big(f_C^1 f_{T_{LW}}^5 - f_C^5 f_{T_{LW}}^1\big) f_w^4 \big(f_K^3 f_L^6 - f_K^6 f_L^3\big) \\
&+ f_C^6 f_{T_{LW}}^1 \Bigg(- f_w^4 f_K^3 f_L^5 + f_w^5 \big(f_K^3 f_L^4 - f_K^4 f_L^3\big)\Bigg) \\
&- f_C^6 f_{T_{LW}}^5 \Bigg(f_w^1 \big(f_K^3 f_L^4 - f_K^4 f_L^3\big) - f_w^4 f_K^3 f_L^1 \Bigg) \\
&- \big(f_C^1 f_{T_{LW}}^5 - f_C^5 f_{T_{LW}}^1\big) \Bigg(f_w^2 \big(f_K^3 f_L^4 - f_K^4 f_L^3\big) + f_w^4 f_K^2 f_L^3 \Bigg).
\end{aligned}
\tag{148}
$$

$$
|D_{W,T_\pi}| = \begin{vmatrix}
0 & f_L^1 & f_w^1 & f_C^1 & 0 & 0 \\
f_K^2 & 0 & f_w^2 & 0 & -1 & f_{T_\pi}^2 \\
f_K^3 & f_L^3 & 0 & 0 & 0 & 0 \\
f_K^4 & f_L^4 & f_w^4 & 0 & 0 & 0 \\
0 & f_L^5 & f_w^5 & f_C^5 & 0 & 0 \\
f_K^6 & f_L^6 & 0 & f_C^6 & -1 & 0
\end{vmatrix}
$$

$$
= \begin{vmatrix}
0 & f_L^1 & f_w^1 & f_C^1 & 0 \\
f_K^3 & f_L^3 & 0 & 0 & 0 \\
f_K^4 & f_L^4 & f_w^4 & 0 & 0 \\
0 & f_L^5 & f_w^5 & f_C^5 & 0 \\
f_K^6 & f_L^6 & 0 & f_C^6 & 0
\end{vmatrix}
+ \begin{vmatrix}
0 & f_L^1 & f_w^1 & f_C^1 & 0 \\
f_K^2 & 0 & f_w^2 & 0 & f_{T_\pi}^2 \\
f_K^3 & f_L^3 & 0 & 0 & 0 \\
f_K^4 & f_L^4 & f_w^4 & 0 & 0 \\
0 & f_L^5 & f_w^5 & f_C^5 & 0
\end{vmatrix}
$$

$$
= -f_C^1 \begin{vmatrix}
f_K^2 & 0 & f_w^2 & f_{T_\pi}^2 \\
f_K^3 & f_L^3 & 0 & 0 \\
f_K^4 & f_L^4 & f_w^4 & 0 \\
0 & f_L^5 & f_w^5 & 0
\end{vmatrix}
- f_C^5 \begin{vmatrix}
0 & f_L^1 & f_w^1 & 0 \\
f_K^2 & 0 & f_w^2 & f_{T_\pi}^2 \\
f_K^3 & f_L^3 & 0 & 0 \\
f_K^4 & f_L^4 & f_w^4 & 0
\end{vmatrix}
$$

$$
= f_C^1 f_{T_\pi}^2 \begin{vmatrix}
f_K^3 & f_L^3 & 0 \\
f_K^4 & f_L^4 & f_w^4 \\
0 & f_L^5 & f_w^5
\end{vmatrix}
- f_C^5 f_{T_\pi}^2 \begin{vmatrix}
0 & f_L^1 & f_w^1 \\
f_K^3 & f_L^3 & 0 \\
f_K^4 & f_L^4 & f_w^4
\end{vmatrix}
$$

$$
\begin{aligned}
&\Leftrightarrow |D_{W,T_\pi}| = f_C^1 f_{T_\pi}^2 \Bigg(- f_w^4 f_K^3 f_L^5 + f_w^5 \big(f_K^3 f_L^4 - f_K^4 f_L^3\big)\Bigg) \\
&- f_C^5 f_{T_\pi}^2 \Bigg(f_w^1 \big(f_K^3 f_L^4 - f_K^4 f_L^3\big) - f_w^4 f_K^3 f_L^1 \Bigg).
\end{aligned}
\tag{149}
$$

$$|D_{W,T_C}| = \begin{vmatrix} 0 & f^1_L & f^1_w & f^1_C & 0 & f^1_{T_C} \\ f^2_K & 0 & f^2_w & 0 & -1 & f^2_{T_C} \\ f^3_K & f^3_L & 0 & 0 & 0 & f^3_{T_C} \\ f^4_K & f^4_L & f^4_w & 0 & 0 & 0 \\ 0 & f^5_L & f^5_w & f^5_C & 0 & f^5_{T_C} \\ f^6_K & f^6_L & 0 & f^6_C & -1 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & f^1_L & f^1_w & f^1_C & f^1_{T_C} \\ f^3_K & f^3_L & 0 & 0 & f^3_{T_C} \\ f^4_K & f^4_L & f^4_w & 0 & 0 \\ 0 & f^5_L & f^5_w & f^5_C & f^5_{T_C} \\ f^6_K & f^6_L & 0 & f^6_C & 0 \end{vmatrix} + \begin{vmatrix} 0 & f^1_L & f^1_w & f^1_C & f^1_{T_C} \\ f^2_K & 0 & f^2_w & 0 & f^2_{T_C} \\ f^3_K & f^3_L & 0 & 0 & f^3_{T_C} \\ f^4_K & f^4_L & f^4_w & 0 & 0 \\ 0 & f^5_L & f^5_w & f^5_C & f^5_{T_C} \end{vmatrix}$$

$$= -f^1_C \begin{vmatrix} f^3_K & f^3_L & 0 & f^3_{T_C} \\ f^4_K & f^4_L & f^4_w & 0 \\ 0 & f^5_L & f^5_w & f^5_{T_C} \\ f^6_K & f^6_L & 0 & 0 \end{vmatrix} + f^5_C \begin{vmatrix} 0 & f^1_L & f^1_w & f^1_{T_C} \\ f^3_K & f^3_L & 0 & f^3_{T_C} \\ f^4_K & f^4_L & f^4_w & 0 \\ f^6_K & f^6_L & 0 & 0 \end{vmatrix}$$

$$-f^6_C \begin{vmatrix} 0 & f^1_L & f^1_w & f^1_{T_C} \\ f^3_K & f^3_L & 0 & f^3_{T_C} \\ f^4_K & f^4_L & f^4_w & 0 \\ 0 & f^5_L & f^5_w & f^5_{T_C} \end{vmatrix} - f^1_C \begin{vmatrix} f^2_K & 0 & f^2_w & f^2_{T_C} \\ f^3_K & f^3_L & 0 & f^3_{T_C} \\ f^4_K & f^4_L & f^4_w & 0 \\ 0 & f^5_L & f^5_w & f^5_{T_C} \end{vmatrix}$$

$$-f^5_C \begin{vmatrix} 0 & f^1_L & f^1_w & f^1_{T_C} \\ f^2_K & 0 & f^2_w & f^2_{T_C} \\ f^3_K & f^3_L & 0 & f^3_{T_C} \\ f^4_K & f^4_L & f^4_w & 0 \end{vmatrix} = f^1_C f^3_{T_C} \begin{vmatrix} f^4_K & f^4_L & f^4_w \\ 0 & f^5_L & f^5_w \\ f^6_K & f^6_L & 0 \end{vmatrix} + f^1_C f^5_{T_C} \begin{vmatrix} f^3_K & f^3_L & 0 \\ f^4_K & f^4_L & f^4_w \\ f^6_K & f^6_L & 0 \end{vmatrix}$$

$$-f^5_C f^1_{T_C} \begin{vmatrix} f^3_K & f^3_L & 0 \\ f^4_K & f^4_L & f^4_w \\ f^6_K & f^6_L & 0 \end{vmatrix} + f^5_C f^3_{T_C} \begin{vmatrix} 0 & f^1_L & f^1_w \\ f^4_K & f^4_L & f^4_w \\ f^6_K & f^6_L & 0 \end{vmatrix} + f^6_C f^1_{T_C} \begin{vmatrix} f^3_K & f^3_L & 0 \\ f^4_K & f^4_L & f^4_w \\ 0 & f^5_L & f^5_w \end{vmatrix}$$

$$-f^6_C f^3_{T_C} \begin{vmatrix} 0 & f^1_L & f^1_w \\ f^4_K & f^4_L & f^4_w \\ 0 & f^5_L & f^5_w \end{vmatrix} - f^6_C f^5_{T_C} \begin{vmatrix} 0 & f^1_L & f^1_w \\ f^3_K & f^3_L & 0 \\ f^4_K & f^4_L & f^4_w \end{vmatrix} + f^1_C f^2_{T_C} \begin{vmatrix} f^3_K & f^3_L & 0 \\ f^4_K & f^4_L & f^4_w \\ 0 & f^5_L & f^5_w \end{vmatrix}$$

$$-f^1_C f^3_{T_C} \begin{vmatrix} f^2_K & 0 & f^2_w \\ f^4_K & f^4_L & f^4_w \\ 0 & f^5_L & f^5_w \end{vmatrix} - f^1_C f^5_{T_C} \begin{vmatrix} f^2_K & 0 & f^2_w \\ f^3_K & f^3_L & 0 \\ f^4_K & f^4_L & f^4_w \end{vmatrix} + f^5_C f^1_{T_C} \begin{vmatrix} f^2_K & 0 & f^2_w \\ f^3_K & f^3_L & 0 \\ f^4_K & f^4_L & f^4_w \end{vmatrix}$$

$$-f_C^5 f_{T_C}^2 \begin{vmatrix} 0 & f_L^1 & f_w^1 \\ f_K^3 & f_L^3 & 0 \\ f_K^4 & f_L^4 & f_w^4 \end{vmatrix} + f_C^5 f_{T_C}^3 \begin{vmatrix} 0 & f_L^1 & f_w^1 \\ f_K^2 & 0 & f_w^2 \\ f_K^4 & f_L^4 & f_w^4 \end{vmatrix}$$

$$= f_C^1 f_{T_C}^3 \begin{vmatrix} f_K^4 & f_L^4 & f_w^4 \\ 0 & f_L^5 & f_w^5 \\ f_K^6 & f_L^6 & 0 \end{vmatrix} + \left(f_C^1 f_{T_C}^5 - f_C^5 f_{T_C}^1\right) \begin{vmatrix} f_K^3 & f_L^3 & 0 \\ f_K^4 & f_L^4 & f_w^4 \\ f_K^6 & f_L^6 & 0 \end{vmatrix}$$

$$+ f_C^5 f_{T_C}^3 \begin{vmatrix} 0 & f_L^1 & f_w^1 \\ f_K^4 & f_L^4 & f_w^4 \\ f_K^6 & f_L^6 & 0 \end{vmatrix} + \left(f_C^6 f_{T_C}^1 + f_C^1 f_{T_C}^2\right) \begin{vmatrix} f_K^3 & f_L^3 & 0 \\ f_K^4 & f_L^4 & f_w^4 \\ 0 & f_L^5 & f_w^5 \end{vmatrix}$$

$$- f_C^6 f_{T_C}^3 \begin{vmatrix} 0 & f_L^1 & f_w^1 \\ f_K^4 & f_L^4 & f_w^4 \\ 0 & f_L^5 & f_w^5 \end{vmatrix} - \left(f_C^6 f_{T_C}^5 + f_C^5 f_{T_C}^2\right) \begin{vmatrix} 0 & f_L^1 & f_w^1 \\ f_K^3 & f_L^3 & 0 \\ f_K^4 & f_L^4 & f_w^4 \end{vmatrix}$$

$$- f_C^1 f_{T_C}^3 \begin{vmatrix} f_K^2 & 0 & f_w^2 \\ f_K^4 & f_L^4 & f_w^4 \\ 0 & f_L^5 & f_w^5 \end{vmatrix} - \left(f_C^1 f_{T_C}^5 - f_C^5 f_{T_C}^1\right) \begin{vmatrix} f_K^2 & 0 & f_w^2 \\ f_K^3 & f_L^3 & 0 \\ f_K^4 & f_L^4 & f_w^4 \end{vmatrix}$$

$$+ f_C^5 f_{T_C}^3 \begin{vmatrix} 0 & f_L^1 & f_w^1 \\ f_K^2 & 0 & f_w^2 \\ f_K^4 & f_L^4 & f_w^4 \end{vmatrix}$$

$$
\begin{aligned}
\Leftrightarrow |D_{W,T_C}| &= f_C^1 f_{T_C}^3 \bigg(- f_w^4 f_K^6 f_L^5 - f_w^5 \big(f_K^4 f_L^6 - f_K^6 f_L^4 \big) \bigg) \\
&- \big(f_C^1 f_{T_C}^5 - f_C^5 f_{T_C}^1 \big) f_w^4 \big(f_K^3 f_L^6 - f_K^6 f_L^3 \big) \\
&+ f_C^5 f_{T_C}^3 \bigg(f_w^1 \big(f_K^4 f_L^6 - f_K^6 f_L^4 \big) + f_w^4 f_K^6 f_L^1 \bigg) \\
&+ \big(f_C^6 f_{T_C}^1 + f_C^1 f_{T_C}^2 \big) \bigg(- f_w^4 f_K^3 f_L^5 + f_w^5 \big(f_K^3 f_L^4 - f_K^4 f_L^3 \big) \bigg) \\
&+ f_C^6 f_{T_C}^3 f_K^4 \big(f_L^1 f_w^5 - f_L^5 f_w^1 \big) \\
&- \big(f_C^6 f_{T_C}^5 + f_C^5 f_{T_C}^2 \big) \bigg(f_w^1 \big(f_K^3 f_L^4 - f_K^4 f_L^3 \big) - f_w^4 f_K^3 f_L^1 \bigg) \\
&- f_C^1 f_{T_C}^3 \bigg(f_K^2 \big(f_L^4 f_w^5 - f_L^5 f_w^4 \big) + f_w^2 f_K^4 f_L^5 \bigg) \\
&- \big(f_C^1 f_{T_C}^5 - f_C^5 f_{T_C}^1 \big) \bigg(f_w^2 \big(f_K^3 f_L^4 - f_K^4 f_L^3 \big) + f_w^4 f_K^2 f_L^3 \bigg) \\
&+ f_C^5 f_{T_C}^3 \bigg(- f_K^2 \big(f_L^1 f_w^4 - f_L^4 f_w^1 \big) + f_K^4 f_L^1 f_w^2 \bigg).
\end{aligned}
\tag{150}
$$

$$
|D_{W,G}| = \begin{vmatrix} 0 & f_L^1 & f_w^1 & f_C^1 & 0 & 0 \\ f_K^2 & 0 & f_w^2 & 0 & -1 & 0 \\ f_K^3 & f_L^3 & 0 & 0 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 & 0 & 0 \\ 0 & f_L^5 & f_w^5 & f_C^5 & 0 & 0 \\ f_K^6 & f_L^6 & 0 & f_C^6 & -1 & f_G^6 \end{vmatrix}
$$

$$
= \begin{vmatrix} 0 & f_L^1 & f_w^1 & f_C^1 & 0 \\ f_K^3 & f_L^3 & 0 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 & 0 \\ 0 & f_L^5 & f_w^5 & f_C^5 & 0 \\ f_K^6 & f_L^6 & 0 & f_C^6 & f_G^6 \end{vmatrix} + \begin{vmatrix} 0 & f_L^1 & f_w^1 & f_C^1 & 0 \\ f_K^2 & 0 & f_w^2 & 0 & 0 \\ f_K^3 & f_L^3 & 0 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 & 0 \\ 0 & f_L^5 & f_w^5 & f_C^5 & 0 \end{vmatrix}
$$

$$
= -f_C^1 \begin{vmatrix} f_K^3 & f_L^3 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 \\ 0 & f_L^5 & f_w^5 & 0 \\ f_K^6 & f_L^6 & 0 & f_G^6 \end{vmatrix} + f_C^5 \begin{vmatrix} 0 & f_L^1 & f_w^1 & 0 \\ f_K^3 & f_L^3 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 \\ f_K^6 & f_L^6 & 0 & f_G^6 \end{vmatrix}
$$

$$-f_C^6 \begin{vmatrix} 0 & f_L^1 & f_w^1 & 0 \\ f_K^3 & f_L^3 & 0 & 0 \\ f_K^4 & f_L^4 & f_w^4 & 0 \\ 0 & f_L^5 & f_w^5 & 0 \end{vmatrix} = -f_C^1 f_G^6 \begin{vmatrix} f_K^3 & f_L^3 & 0 \\ f_K^4 & f_L^4 & f_w^4 \\ 0 & f_L^5 & f_w^5 \end{vmatrix}$$

$$+f_C^5 f_G^6 \begin{vmatrix} 0 & f_L^1 & f_w^1 \\ f_K^3 & f_L^3 & 0 \\ f_K^4 & f_L^4 & f_w^4 \end{vmatrix}$$

$$\begin{aligned} \Leftrightarrow |D_{W,G}| = -f_C^1 f_G^6 \Big(- f_w^4 f_K^3 f_L^5 + f_w^5 \big(f_K^3 f_L^4 - f_K^4 f_L^3 \big) \Big) \\ + f_C^5 f_G^6 \Big(f_w^1 \big(f_K^3 f_L^4 - f_K^4 f_L^3 \big) - f_w^4 f_K^3 f_L^1 \Big). \end{aligned} \tag{151}$$

D.2.3 Calibration values

Certain parameters and endogenous variables are to be calibrated. These are listed in tables 1 and 2 below.

Calibration of endogenous variables, the period length being one year:

Table 1: Variable values

Variable	Description	Value	Unit
G^*	public expenditures	1.19221×10^{11}	EUR
K^*	firm capital stock	2.61×10^{11}	EUR
w^*	gross wage rate	39822	EUR
L^*	entrepreneurs' labour demand	2447250	persons
N	workers' labour supply	2682250	persons

The values reported in table 1 are based on data from Statistics Finland[16].

[16] Statistics Finland. `https://www.stat.fi/index_en.html`. Accessed: February 2018.

Calibration of parameters:

Table 2: Parameter values

Parameter	Description	Value
ρ	rate of time preference	0.0638
δ	depreciation rate	0.05
σ	factor elasticity	0.6735
η	income share of capital	0.423
α	ratio between unemployment benefits and wage rate	0.5
T_{LW}	worker's labour tax	0.3
T_{π}	profit tax rate	0.2
T_C	consumption tax rate	0.24

The values for parameters ρ, δ, σ and η, reported in table 2 are based on sources from the literature[17]. The value for α is based on data from Statistics Finland[18] and The Federation of Unemployment Funds in Finland[19]. The values for the tax rates T_{LW}, T_{π} and T_C are based on information from the Taxpayers Association of Finland[20] and the Finnish Tax Administration[21,22].

Based on the system of equations discussed in chapter $D.1$ and the values reported in tables 1 and 2, the following calibrations are done:

[17] Alvarez-Cuadrado, Francisco; Ngo, Van Long and Poschke, Markus. Capital Labor Substitution, Structural Change, and the Labor Income Share. CIRANO - Scientific Publications 2014 nr 2. 2014.

[18] Statistics Finland, ibid.

[19] The Federation of Unemployment Funds in Finland. http://www.tyj.fi/eng/earnings-related_allowance/allowance_calculator/. Accessed: February 2018.

[20] Taxpayers Association of Finland.
https://www.veronmaksajat.fi/Vieraskieliset-sivut/Taxpayers-Association-of-Finland-TAF/.
Value based on table published in https://www.veronmaksajat.fi/luvut/Laskelmat/Palkansaajan-veroprosentit/. Accessed: February 2018.

[21] Finnish Tax Administration.
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Table 3: Calibrated values

Symbol	Description	Value	Unit
A	production level parameter	1823260.741	1
P	price level	0.007082512	EUR
π	entrepreneurs' profits	117047708.7	EUR
C	workers' consumption	1.36479×10^{13}	EUR
W	workers' wealth/savings	7.57746×10^{11}	EUR
ψ	worker's dis-utility of working	0.027099415	1

D.2.4 Connections between some derivatives and variables

In addition to the results in the chapter above, below follows some calculations in order to develop further tools, needed for the determinants.

First of all, since the production function is assumed to be concave, it is known that

$$\begin{aligned} &F_K(K^*, L^*) > 0, \quad F_L(K^*, L^*) > 0, \\ &F_{KK}(K^*, L^*) < 0, \quad F_{LL}(K^*, L^*) < 0, \\ &F_{LK}(K^*, L^*) > 0, \quad F_{KL}(K^*, L^*) > 0. \end{aligned} \tag{152}$$

Based on equation (48), equation f^4 in the equation system in chapter $D.1$ can be rewritten as

$$F_L(K^*, L^*) = \frac{w^*}{P^*}. \tag{153}$$

Based on equation (41) and equation f^3 in the system of equations presented in appendix $D.1$, it holds that

$$F_K(K^*, L^*) = (1 + T_C^*)(\rho + \delta). \tag{154}$$

Based on equations (12), (41) and (48), it holds that

$$\begin{aligned}
&\frac{F_K(K^*,L^*)K^*}{F(K^*,L^*)}+\frac{F_L(K^*,L^*)L^*}{F(K^*,L^*)}=\\
&\frac{A\eta[\eta(K^*)^{\frac{\sigma-1}{\sigma}}+(1-\eta)(L^*)^{\frac{\sigma-1}{\sigma}}]^{\frac{1}{\sigma-1}}(K^*)^{-\frac{1}{\sigma}}K^*}{A[\eta(K^*)^{\frac{\sigma-1}{\sigma}}+(1-\eta)(L^*)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}}\\
&+\frac{A(1-\eta)[\eta(K^*)^{\frac{\sigma-1}{\sigma}}+(1-\eta)(L^*)^{\frac{\sigma-1}{\sigma}}]^{\frac{1}{\sigma-1}}(L^*)^{-\frac{1}{\sigma}}L^*}{A[\eta(K^*)^{\frac{\sigma-1}{\sigma}}+(1-\eta)(L^*)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}}\\
&=\frac{[\eta(K^*)^{\frac{\sigma-1}{\sigma}}+(1-\eta)(L^*)^{\frac{\sigma-1}{\sigma}}]^{\frac{1}{\sigma-1}}[\eta(K^*)^{\frac{\sigma-1}{\sigma}}+(1-\eta)(L^*)^{\frac{\sigma-1}{\sigma}}]}{[\eta(K^*)^{\frac{\sigma-1}{\sigma}}+(1-\eta)(L^*)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}}=1\\
&\Leftrightarrow\frac{F_K(K^*,L^*)K^*}{F(K^*,L^*)}+\frac{F_L(K^*,L^*)L^*}{F(K^*,L^*)}=1.
\end{aligned}\tag{155}$$

Now, based on equation f^2 in the equation system in appendix $D.1$ and equations (12), (153), (154) and (155), it holds that

$$\begin{aligned}
&\pi^*=(1-T^*_\pi)\big[P^*F(K^*,L^*)-w^*L^*-(1+T^*_C)P^*\delta K^*\big]\\
&\Leftrightarrow\pi^*=(1-T^*_\pi)\big[P^*\big(F_K(K^*,L^*)K^*+F_L(K^*,L^*)L^*\big)\\
&-w^*L^*-(1+T^*_C)P^*\delta K^*\big]\\
&\Leftrightarrow\pi^*=(1-T^*_\pi)\big[P^*\big((1+T^*_C)(\rho+\delta)K^*+\frac{w^*L^*}{P^*}\big)\\
&-w^*L^*-(1+T^*_C)P^*\delta K^*\big]\\
&\Leftrightarrow\pi^*=(1-T^*_\pi)\big[P^*(1+T^*_C)(\rho+\delta)K^*+w^*L^*\\
&-w^*L^*-(1+T^*_C)P^*\delta K^*\big]\\
&\Leftrightarrow\pi^*=(1-T^*_\pi)P^*(1+T^*_C)\rho K^*.
\end{aligned}\tag{156}$$

D.2.5 Cramer's rule calibrated

Applying Cramer's rule, expressed by equation (66), while considering equations (67) - (151) and the values mentioned in tables 1, 2 and 3, gives the following equations:

$$
\begin{aligned}
&dK^* = -\frac{|D_{K,T_{LW}}|}{|D|}dT_{LW}^* - \frac{|D_{K,T_\pi}|}{|D|}dT_\pi^* - \frac{|D_{K,T_C}|}{|D|}dT_C^* - \frac{|D_{K,G}|}{|D|}dG^*,\\
&\text{where} \quad -\frac{|D_{K,T_{LW}}|}{|D|} = -32630733980, \quad -\frac{|D_{K,T_\pi}|}{|D|} = -34573593.07,\\
&-\frac{|D_{K,T_C}|}{|D|} = -1.60582 \times 10^{11}, \quad -\frac{|D_{K,G}|}{|D|} = 0.001673628.
\end{aligned} \tag{157}
$$

$$
\begin{aligned}
&dL^* = -\frac{|D_{L,T_{LW}}|}{|D|}dT_{LW}^* - \frac{|D_{L,T_\pi}|}{|D|}dT_\pi^* - \frac{|D_{L,T_C}|}{|D|}dT_C^* - \frac{|D_{L,G}|}{|D|}dG^*,\\
&\text{where} \quad -\frac{|D_{L,T_{LW}}|}{|D|} = -305960.0143, \quad -\frac{|D_{L,T_\pi}|}{|D|} = -324.1771097,\\
&-\frac{|D_{L,T_C}|}{|D|} = -172914.4443, \quad -\frac{|D_{L,G}|}{|D|} = 1.56927 \times 10^{-8}.
\end{aligned} \tag{158}
$$

$$
\begin{aligned}
&dw^* = -\frac{|D_{w,T_{LW}}|}{|D|}dT_{LW}^* - \frac{|D_{w,T_\pi}|}{|D|}dT_\pi^* - \frac{|D_{w,T_C}|}{|D|}dT_C^* - \frac{|D_{w,G}|}{|D|}dG^*,\\
&\text{where} \quad -\frac{|D_{w,T_{LW}}|}{|D|} = -1.64163 \times 10^{-11},\\
&\quad -\frac{|D_{w,T_\pi}|}{|D|} = -1.73937 \times 10^{-14}, \quad -\frac{|D_{w,T_C}|}{|D|} = -85.98158256,\\
&-\frac{|D_{w,G}|}{|D|} = 8.41989 \times 10^{-25}.
\end{aligned} \tag{159}
$$

$$
\begin{aligned}
&dC^* = -\frac{|D_{C,T_{LW}}|}{|D|}dT_{LW}^* - \frac{|D_{C,T_\pi}|}{|D|}dT_\pi^* - \frac{|D_{C,T_C}|}{|D|}dT_C^* - \frac{|D_{C,G}|}{|D|}dG^*,\\
&\text{where} \quad -\frac{|D_{C,T_{LW}}|}{|D|} = -1.72119 \times 10^{12}, \quad -\frac{|D_{C,T_\pi}|}{|D|} = 18834198077,\\
&-\frac{|D_{C,T_C}|}{|D|} = -9.90022 \times 10^{11}, \quad -\frac{|D_{C,G}|}{|D|} = -0.911720131.
\end{aligned} \tag{160}
$$

$$
\begin{aligned}
&d\pi^* = -\frac{|D_{\pi,T_{LW}}|}{|D|}dT^*_{LW} - \frac{|D_{\pi,T_\pi}|}{|D|}dT^*_\pi - \frac{|D_{\pi,T_C}|}{|D|}dT^*_C - \frac{|D_{\pi,G}|}{|D|}dG^*,\\
&\text{where} \quad -\frac{|D_{\pi,T_{LW}}|}{|D|} = -14633535.04, \quad -\frac{|D_{\pi,T_\pi}|}{|D|} = -146325140.7, \\
&-\frac{|D_{\pi,T_C}|}{|D|} = 22379010.74, \quad -\frac{|D_{\pi,G}|}{|D|} = -7.50553 \times 10^{-7}.
\end{aligned} \tag{161}
$$

$$
\begin{aligned}
&dW^* = -\frac{|D_{W,T_{LW}}|}{|D|}dT^*_{LW} - \frac{|D_{W,T_\pi}|}{|D|}dT^*_\pi - \frac{|D_{W,T_C}|}{|D|}dT^*_C - \frac{|D_{W,G}|}{|D|}dG^*,\\
&\text{where} \quad -\frac{|D_{W,T_{LW}}|}{|D|} = 1.43008 \times 10^{12}, \quad -\frac{|D_{W,T_\pi}|}{|D|} = 2662176340, \\
&-\frac{|D_{W,T_C}|}{|D|} = 1.41832 \times 10^{12}, \quad -\frac{|D_{W,G}|}{|D|} = -0.128869822.
\end{aligned} \tag{162}
$$

D.3 Welfare analysis

In preparation for the welfare analysis, it is necessary to develop the steady state version of the government's budget constraint (16). Taking into consideration equation (17), this is done in the following calculation:

$$
\begin{aligned}
&T_{\pi,t}\big[P_tF(K_t, L_t) - w_tL_t - (1 + T_{C,t})P_t\big(K_{t+1} - (1-\delta)K_t\big)\big]\\
&+ T_{C,t}P_t\big(C_t + K_{t+1} - (1-\delta)K_t\big) + T_{LW,t}w_tL_t\\
&+ D_{t+1} - (1 + r_t)D_t - P_tG_t - (N - L_t)\alpha w_t(1 - T_{LW,t}) = 0
\end{aligned}
$$

$$
\begin{aligned}
&\Leftrightarrow T^*_\pi\big[P^*F(K^*, L^*) - w^*L^* - (1 + T^*_C)P^*\big(K^* - (1-\delta)K^*\big)\big]\\
&+ T^*_CP^*\big(C^* + K^* - (1-\delta)K^*\big) + T^*_{LW}w^*L^*\\
&+ D^* - (1 + r^*)D^* - P^*G^* - (N - L^*)\alpha w^*(1 - T^*_{LW}) = 0
\end{aligned}
$$

$$
\begin{aligned}
&\Leftrightarrow T^*_\pi\big[P^*F(K^*, L^*) - w^*L^* - (1 + T^*_C)P^*\delta K^*\big] + T^*_CP^*\big(C^* + \delta K^*\big)\\
&+ T^*_{LW}w^*L^* - r^*D^* - P^*G^* - (N - L^*)\alpha w^*(1 - T^*_{LW}) = 0.
\end{aligned} \tag{163}
$$

Equation (163) expresses the steady state version of the government's budget constraint.

Totally differentiating the steady state budget (163) with regards to T_{LW}^*, T_π^*, T_C^* and G^* generates:

$$\begin{aligned} & w^*\big(L^* + (N - L^*)\alpha\big)dT_{LW}^* \\ & + \big[P^*F(K^*, L^*) - w^*L^* - (1 + T_C^*)P^*\delta K^*\big]dT_\pi^* \\ & + \big(- T_\pi^* P^*\delta K^* + P^*(C^* + \delta K^*)\big)dT_C^* - P^*dG^* = 0. \end{aligned} \tag{164}$$

Based on equation (164), the connection between changes in different taxation and public expenditure parameters can be calculated.

In order to be able to analyse how the utility of workers change in steady state as a consequence of tax rate changes, the utility function (1) must first be expressed in steady state terms. Substituting steady state values into (1) gives

$$U_W^* \doteq U_W(C^*, L^*) = \log(C^*) + \psi \log(N - L^*). \tag{165}$$

Totally differentiating equation (165) with regard to U_W^*, C^* and L^*, generates

$$dU_W^* = \frac{1}{C^*}dC^* - \frac{\psi}{N - L^*}dL^*. \tag{166}$$

In order to be able to analyse how the utility of entrepreneurs change in steady state as a consequence of tax rate changes, the utility function (6) must first be expressed in steady state terms. Substituting steady state values into (6) gives

$$U_E^* \doteq U_E(\pi^*) = \log(\pi^*). \tag{167}$$

Totally differentiating equation (167) with regard to U_E^* and π^*, generates

$$dU_E^* = \frac{1}{\pi^*}d\pi^*. \tag{168}$$

Now, based on the above calculations, it is possible to assess the different welfare effects of tax reform proposals. In order to do that

Based on equation (166) and values mentioned in the tables 1 and 2, it holds that

$$\frac{\partial U_W^*}{\partial C^*} > 0 \quad \text{and} \quad \frac{\partial U_W^*}{\partial L^*} < 0. \tag{169}$$

Based on equation (168) and values mentioned in the tables 1 and 2, it holds that

$$\frac{\partial U_E^*}{\partial \pi^*} > 0. \tag{170}$$

Further it is possible to investigate the effects of different tax reforms for instance on the workers' consumption, the entrepreneurs' profits, the wage rate, the employment rate being defined as $E^* = \frac{L^*}{N}$ and the capital stock.

Additionally, the tax effects on the ratio between the entrepreneurs' profits and the value of the production as well as on the ratio between the workers' labour income and the value of the production can be examined. In order to do that some calculations are necessary to conduct.

Totally differentiating equation f^3 in the equation system of chapter $D.1$ with regard to A, K^*, L^* and T_C^*, keeping in mind equations (41), (44), (55),

(157) and (158), leads to

$$\frac{F_K(K^*,L^*)}{A}dA + F_{KK}(K^*,L^*)dK^* + F_{LK}(K^*,L^*)dL^* - (\rho+\delta)dT_C^* = 0$$
$$\Leftrightarrow \frac{F_K(K^*,L^*)}{A}dA + \Big(- F_{KK}(K^*,L^*)\frac{|D_{K,T_{LW}}|}{|D|} - F_{LK}(K^*,L^*)\frac{|D_{L,T_{LW}}|}{|D|}\Big)dT_{LW}^*$$
$$+ \Big(- F_{KK}(K^*,L^*)\frac{|D_{K,T_\pi}|}{|D|} - F_{LK}(K^*,L^*)\frac{|D_{L,T_\pi}|}{|D|}\Big)dT_\pi^*$$
$$+ \Big(- F_{KK}(K^*,L^*)\frac{|D_{K,T_C}|}{|D|} - F_{LK}(K^*,L^*)\frac{|D_{L,T_C}|}{|D|} - (\rho+\delta)\Big)dT_C^*$$
$$+ \Big(- F_{KK}(K^*,L^*)\frac{|D_{K,G}|}{|D|} - F_{LK}(K^*,L^*)\frac{|D_{L,G}|}{|D|}\Big)dG^* = 0$$

$$\begin{aligned}
&\Leftrightarrow dA = -\frac{A}{F_K(K^*,L^*)} \\
&\times \Big(- F_{KK}(K^*,L^*)\frac{|D_{K,T_{LW}}|}{|D|} - F_{LK}(K^*,L^*)\frac{|D_{L,T_{LW}}|}{|D|}\Big)dT_{LW}^* \\
&- \frac{A}{F_K(K^*,L^*)}\Big(- F_{KK}(K^*,L^*)\frac{|D_{K,T_\pi}|}{|D|} - F_{LK}(K^*,L^*)\frac{|D_{L,T_\pi}|}{|D|}\Big)dT_\pi^* \\
&- \frac{A}{F_K(K^*,L^*)}\Big(- F_{KK}(K^*,L^*)\frac{|D_{K,T_C}|}{|D|} \\
&- F_{LK}(K^*,L^*)\frac{|D_{L,T_C}|}{|D|} - (\rho+\delta)\Big)dT_C^* \\
&- \frac{A}{F_K(K^*,L^*)}\Big(- F_{KK}(K^*,L^*)\frac{|D_{K,G}|}{|D|} - F_{LK}(K^*,L^*)\frac{|D_{L,G}|}{|D|}\Big)dG^*.
\end{aligned} \tag{171}$$

Totally differentiating the steady state version of equation (12) with regard to $F(K^*,L^*)$, A, K^* and L^*, taking into consideration equations (41), (48), (157), (158) and (171), leads to

$$dF(K^*,L^*) = \frac{F(K^*,L^*)}{A}dA + F_K(K^*,L^*)dK^* + F_L(K^*,L^*)dL^*$$

$$
\begin{aligned}
&\Leftrightarrow dF(K^*,L^*) = \Bigg[- \frac{F(K^*,L^*)}{F_K(K^*,L^*)} \\
&\times \Bigg(- F_{KK}(K^*,L^*)\frac{|D_{K,T_{LW}}|}{|D|} - F_{LK}(K^*,L^*)\frac{|D_{L,T_{LW}}|}{|D|}\Bigg) \\
&- F_K(K^*,L^*)\frac{|D_{K,T_{LW}}|}{|D|} - F_L(K^*,L^*)\frac{|D_{L,T_{LW}}|}{|D|}\Bigg] dT^*_{LW} \\
&+ \Bigg[- \frac{F(K^*,L^*)}{F_K(K^*,L^*)}\Bigg(- F_{KK}(K^*,L^*)\frac{|D_{K,T_\pi}|}{|D|} - F_{LK}(K^*,L^*)\frac{|D_{L,T_\pi}|}{|D|}\Bigg) \\
&- F_K(K^*,L^*)\frac{|D_{K,T_\pi}|}{|D|} - F_L(K^*,L^*)\frac{|D_{L,T_\pi}|}{|D|}\Bigg] dT^*_\pi \\
&+ \Bigg[- \frac{F(K^*,L^*)}{F_K(K^*,L^*)}\Bigg(- F_{KK}(K^*,L^*)\frac{|D_{K,T_C}|}{|D|} \\
&- F_{LK}(K^*,L^*)\frac{|D_{L,T_C}|}{|D|} - (\rho+\delta)\Bigg) - F_K(K^*,L^*)\frac{|D_{K,T_C}|}{|D|} \\
&- F_L(K^*,L^*)\frac{|D_{L,T_C}|}{|D|}\Bigg] dT^*_C \\
&+ \Bigg[- \frac{F(K^*,L^*)}{F_K(K^*,L^*)}\Bigg(- F_{KK}(K^*,L^*)\frac{|D_{K,G}|}{|D|} - F_{LK}(K^*,L^*)\frac{|D_{L,G}|}{|D|}\Bigg) \\
&- F_K(K^*,L^*)\frac{|D_{K,G}|}{|D|} - F_L(K^*,L^*)\frac{|D_{L,G}|}{|D|}\Bigg] dG^*.
\end{aligned} \tag{172}
$$

Totally differentiating equation f^4 in the equation system of chapter $D.1$ with regard to P^*, K^*, L^*, A and w^*, keeping in mind equations (48), (54), (55), (157), (158), (159) and (171), leads to

$$
\begin{aligned}
&F_L(K^*,L^*)dP^* + P^*F_{LK}(K^*,L^*)dK^* + P^*F_{LL}(K^*,L^*)dL^* \\
&+ P^*\frac{F_L(K^*,L^*)}{A}dA - dw^* = 0
\end{aligned}
$$

$$
\begin{aligned}
&\Leftrightarrow F_L(K^*,L^*)dP^* + \Bigg(- P^*F_{LK}(K^*,L^*)\frac{|D_{K,T_{LW}}|}{|D|} - P^*F_{LL}(K^*,L^*)\frac{|D_{L,T_{LW}}|}{|D|} \\
&+ P^*\frac{F_L(K^*,L^*)}{A}\frac{\partial A}{\partial T^*_{LW}} + \frac{|D_{w,T_{LW}}|}{|D|}\Bigg)dT^*_{LW} \\
&+ \Bigg(- P^*F_{LK}(K^*,L^*)\frac{|D_{K,T_\pi}|}{|D|} - P^*F_{LL}(K^*,L^*)\frac{|D_{L,T_\pi}|}{|D|} \\
&+ P^*\frac{F_L(K^*,L^*)}{A}\frac{\partial A}{\partial T^*_{\pi}} + \frac{|D_{w,T_\pi}|}{|D|}\Bigg)dT^*_{\pi} \\
&+ \Bigg(- P^*F_{LK}(K^*,L^*)\frac{|D_{K,T_C}|}{|D|} - P^*F_{LL}(K^*,L^*)\frac{|D_{L,T_C}|}{|D|} \\
&+ P^*\frac{F_L(K^*,L^*)}{A}\frac{\partial A}{\partial T^*_{C}} + \frac{|D_{w,T_C}|}{|D|}\Bigg)dT^*_{C} \\
&+ \Bigg(- P^*F_{LK}(K^*,L^*)\frac{|D_{K,G}|}{|D|} - P^*F_{LL}(K^*,L^*)\frac{|D_{L,G}|}{|D|} \\
&+ P^*\frac{F_L(K^*,L^*)}{A}\frac{\partial A}{\partial G^*} + \frac{|D_{w,G}|}{|D|}\Bigg)dG^* = 0
\end{aligned}
$$

$$
\begin{aligned}
&\Leftrightarrow dP^* = -\Bigg(- \frac{P^*F_{LK}(K^*,L^*)}{F_L(K^*,L^*)}\frac{|D_{K,T_{LW}}|}{|D|} \\
&- \frac{P^*F_{LL}(K^*,L^*)}{F_L(K^*,L^*)}\frac{|D_{L,T_{LW}}|}{|D|} + \frac{P^*}{A}\frac{\partial A}{\partial T^*_{LW}} \\
&+ \frac{1}{F_L(K^*,L^*)}\frac{|D_{w,T_{LW}}|}{|D|}\Bigg)dT^*_{LW} \\
&- \Bigg(- \frac{P^*F_{LK}(K^*,L^*)}{F_L(K^*,L^*)}\frac{|D_{K,T_\pi}|}{|D|} - \frac{P^*F_{LL}(K^*,L^*)}{F_L(K^*,L^*)}\frac{|D_{L,T_\pi}|}{|D|} \\
&+ \frac{P^*}{A}\frac{\partial A}{\partial T^*_{\pi}} + \frac{1}{F_L(K^*,L^*)}\frac{|D_{w,T_\pi}|}{|D|}\Bigg)dT^*_{\pi} \\
&- \Bigg(- \frac{P^*F_{LK}(K^*,L^*)}{F_L(K^*,L^*)}\frac{|D_{K,T_C}|}{|D|} - \frac{P^*F_{LL}(K^*,L^*)}{F_L(K^*,L^*)}\frac{|D_{L,T_C}|}{|D|} \\
&+ \frac{P^*}{A}\frac{\partial A}{\partial T^*_{C}} + \frac{1}{F_L(K^*,L^*)}\frac{|D_{w,T_C}|}{|D|}\Bigg)dT^*_{C} \\
&- \Bigg(- \frac{P^*F_{LK}(K^*,L^*)}{F_L(K^*,L^*)}\frac{|D_{K,G}|}{|D|} - \frac{P^*F_{LL}(K^*,L^*)}{F_L(K^*,L^*)}\frac{|D_{L,G}|}{|D|} \\
&+ \frac{P^*}{A}\frac{\partial A}{\partial G^*} + \frac{1}{F_L(K^*,L^*)}\frac{|D_{w,G}|}{|D|}\Bigg)dG^*
\end{aligned} \tag{173}
$$

The ratio between the entrepreneurs' profits and the value of the production is defined as

$$\frac{\pi^*}{P^*F(K^*,L^*)} \doteq \Omega^*_\pi \tag{174}$$

Totally differentiating equation (174) with regard to Ω^*_π, π^*, P^* and $F(K^*,L^*)$, keeping in mind equations (12), (161), (172) and (173), leads to

$$\begin{aligned}
d\Omega^*_\pi &= \frac{1}{P^*F(K^*,L^*)}d\pi^* - \frac{\pi^*F(K^*,L^*)}{\big(P^*F(K^*,L^*)\big)^2}dP^* - \frac{\pi^*P^*}{\big(P^*F(K^*,L^*)\big)^2}dF(K^*,L^*) \\
&\Leftrightarrow d\Omega^*_\pi = \bigg(- \frac{1}{P^*F(K^*,L^*)}\frac{|D_{\pi,T_{LW}}|}{|D|} - \frac{\pi^*F(K^*,L^*)}{\big(P^*F(K^*,L^*)\big)^2}\frac{\partial P^*}{\partial T^*_{LW}} \\
&- \frac{\pi^*P^*}{\big(P^*F(K^*,L^*)\big)^2}\frac{\partial F(K^*,L^*)}{\partial T^*_{LW}}\bigg)dT^*_{LW} \\
&+ \bigg(- \frac{1}{P^*F(K^*,L^*)}\frac{|D_{\pi,T_\pi}|}{|D|} - \frac{\pi^*F(K^*,L^*)}{\big(P^*F(K^*,L^*)\big)^2}\frac{\partial P^*}{\partial T^*_\pi} \\
&- \frac{\pi^*P^*}{\big(P^*F(K^*,L^*)\big)^2}\frac{\partial F(K^*,L^*)}{\partial T^*_\pi}\bigg)dT^*_\pi \\
&+ \bigg(- \frac{1}{P^*F(K^*,L^*)}\frac{|D_{\pi,T_C}|}{|D|} - \frac{\pi^*F(K^*,L^*)}{\big(P^*F(K^*,L^*)\big)^2}\frac{\partial P^*}{\partial T^*_C} \\
&- \frac{\pi^*P^*}{\big(P^*F(K^*,L^*)\big)^2}\frac{\partial F(K^*,L^*)}{\partial T^*_C}\bigg)dT^*_C \\
&+ \bigg(- \frac{1}{P^*F(K^*,L^*)}\frac{|D_{\pi,G}|}{|D|} - \frac{\pi^*F(K^*,L^*)}{\big(P^*F(K^*,L^*)\big)^2}\frac{\partial P^*}{\partial G^*} \\
&- \frac{\pi^*P^*}{\big(P^*F(K^*,L^*)\big)^2}\frac{\partial F(K^*,L^*)}{\partial G^*}\bigg)dG^*.
\end{aligned} \tag{175}$$

The ratio between the workers' labour income and the value of the production is defined as

$$\frac{L^*w^*}{P^*F(K^*,L^*)} \doteq \Omega^*_{Lw} \tag{176}$$

Totally differentiating equation (176) with regard to Ω^*_{Lw}, L^*, w^*, P^* and $F(K^*, L^*)$, keeping in mind equations (12), (48), (158), (159), (172) and (173), leads to

$$\begin{aligned}
&d\Omega^*_{Lw} = \left(\frac{w^*}{P^*F(K^*,L^*)} - \frac{L^*w^*P^*F_L(K^*,L^*)}{\big(P^*F(K^*,L^*)\big)^2}\right)dL^* + \frac{L^*}{P^*F(K^*,L^*)}dw^* \\
&- \frac{L^*w^*F(K^*,L^*)}{\big(P^*F(K^*,L^*)\big)^2}dP^* - \frac{L^*w^*P^*}{\big(P^*F(K^*,L^*)\big)^2}dF(K^*,L^*) \\
&\Leftrightarrow d\Omega^*_{Lw} = \Bigg[-\left(\frac{w^*}{P^*F(K^*,L^*)} - \frac{L^*w^*P^*F_L(K^*,L^*)}{\big(P^*F(K^*,L^*)\big)^2}\right)\frac{|D_{L,T_{LW}}|}{|D|} \\
&- \frac{L^*}{P^*F(K^*,L^*)}\frac{|D_{w,T_{LW}}|}{|D|} - \frac{L^*w^*F(K^*,L^*)}{\big(P^*F(K^*,L^*)\big)^2}\frac{\partial P^*}{\partial T^*_{LW}} \\
&- \frac{L^*w^*P^*}{\big(P^*F(K^*,L^*)\big)^2}\frac{\partial F(K^*,L^*)}{\partial T^*_{LW}}\Bigg]dT^*_{LW} \\
&+\Bigg[-\left(\frac{w^*}{P^*F(K^*,L^*)} - \frac{L^*w^*P^*F_L(K^*,L^*)}{\big(P^*F(K^*,L^*)\big)^2}\right)\frac{|D_{L,T_\pi}|}{|D|} \\
&- \frac{L^*}{P^*F(K^*,L^*)}\frac{|D_{w,T_\pi}|}{|D|} - \frac{L^*w^*F(K^*,L^*)}{\big(P^*F(K^*,L^*)\big)^2}\frac{\partial P^*}{\partial T^*_\pi} \\
&- \frac{L^*w^*P^*}{\big(P^*F(K^*,L^*)\big)^2}\frac{\partial F(K^*,L^*)}{\partial T^*_\pi}\Bigg]dT^*_\pi \\
&+\Bigg[-\left(\frac{w^*}{P^*F(K^*,L^*)} - \frac{L^*w^*P^*F_L(K^*,L^*)}{\big(P^*F(K^*,L^*)\big)^2}\right)\frac{|D_{L,T_C}|}{|D|} \\
&- \frac{L^*}{P^*F(K^*,L^*)}\frac{|D_{w,T_C}|}{|D|} - \frac{L^*w^*F(K^*,L^*)}{\big(P^*F(K^*,L^*)\big)^2}\frac{\partial P^*}{\partial T^*_C} \\
&- \frac{L^*w^*P^*}{\big(P^*F(K^*,L^*)\big)^2}\frac{\partial F(K^*,L^*)}{\partial T^*_C}\Bigg]dT^*_C \\
&+\Bigg[-\left(\frac{w^*}{P^*F(K^*,L^*)} - \frac{L^*w^*P^*F_L(K^*,L^*)}{\big(P^*F(K^*,L^*)\big)^2}\right)\frac{|D_{L,G}|}{|D|} \\
&- \frac{L^*}{P^*F(K^*,L^*)}\frac{|D_{w,G}|}{|D|} - \frac{L^*w^*F(K^*,L^*)}{\big(P^*F(K^*,L^*)\big)^2}\frac{\partial P^*}{\partial G^*} \\
&- \frac{L^*w^*P^*}{\big(P^*F(K^*,L^*)\big)^2}\frac{\partial F(K^*,L^*)}{\partial G^*}\Bigg]dG^*.
\end{aligned} \tag{177}$$

Based on the calculations above, the effect of tax reforms on the gross income of the entrepreneur can be investigated. The gross income, as expressed in equation (11), is

$$P^* F(K^*, L^*) \doteq Y^*. \tag{178}$$

Totally differentiating equation (178) with regard to Y^*, P^* and $F(K^*, L^*)$, keeping in mind equations (172) and (173), leads to

$$\begin{aligned}
dY^* &= F(K^*, L^*)dP^* + P^* dF(K^*, L^*) \\
\Leftrightarrow dY^* &= \left(F(K^*, L^*) \frac{\partial P^*}{\partial T^*_{Lw}} + P^* \frac{\partial F(K^*, L^*)}{\partial T^*_{Lw}} \right) dT^*_{Lw} \\
&+ \left(F(K^*, L^*) \frac{\partial P^*}{\partial T^*_{\pi}} + P^* \frac{\partial F(K^*, L^*)}{\partial T^*_{\pi}} \right) dT^*_{\pi} \\
&+ \left(F(K^*, L^*) \frac{\partial P^*}{\partial T^*_{C}} + P^* \frac{\partial F(K^*, L^*)}{\partial T^*_{C}} \right) dT^*_{C} \\
&+ \left(F(K^*, L^*) \frac{\partial P^*}{\partial G^*} + P^* \frac{\partial F(K^*, L^*)}{\partial G^*} \right) dG^*.
\end{aligned} \tag{179}$$

D.3.1 Tax reform: Shifting the taxation from labour to consumption

Assume that the profit taxation and the public expenditures are kept constant. Thus when the worker's labour taxation changes, the tax rate on consumption must change in order to keep the government's budget balanced.

Based on equations (164) and (165), the effect of this reform proposal on the utility of a worker can be estimated. The change in the worker's labour taxation has a direct effect on the utility as well as an indirect effect, generated by the change in consumption taxation. The total effect on the utility is

$$
\begin{aligned}
&\frac{\partial U_W^*}{\partial T_{LW}^*}\Big|_{dT_\pi^*=dG^*=0} = \frac{\partial U_W^*}{\partial T_{LW}^*} + \frac{\partial T_C^*}{\partial T_{LW}^*}\frac{\partial U_W^*}{\partial T_C^*} \\
&= \frac{1}{C^*}\frac{\partial C^*}{\partial T_{LW}^*} - \frac{\psi}{N-L^*}\frac{\partial L^*}{\partial T_{LW}^*} \\
&- \frac{w^*\big(L^*+(N-L^*)\alpha\big)}{-T_\pi^* P^*\delta K^* + P^*(C^*+\delta K^*)}\left(\frac{1}{C^*}\frac{\partial C^*}{\partial T_C^*} - \frac{\psi}{N-L^*}\frac{\partial L^*}{\partial T_C^*}\right).
\end{aligned}
\tag{180}
$$

Based on equations (158), (160) and on the values mentioned in tables 1, 2 and 3, equation (180) is negative. If the worker's labour tax rate is decreased by one percentage point, leading to an increased consumption tax rate, the worker's utility increases.

Correspondingly, regarding the entrepreneurs, the total effect on the utility, based on equations (164) and (167), is

$$
\begin{aligned}
&\frac{\partial U_E^*}{\partial T_{LW}^*}\Big|_{dT_\pi^*=dG^*=0} = \frac{\partial U_E^*}{\partial T_{LW}^*} + \frac{\partial T_C^*}{\partial T_{LW}^*}\frac{\partial U_E^*}{\partial T_C^*} \\
&= \frac{1}{\pi^*}\frac{\partial \pi^*}{\partial T_{LW}^*} - \frac{w^*\big(L^*+(N-L^*)\alpha\big)}{-T_\pi^* P^*\delta K^* + P^*(C^*+\delta K^*)}\frac{1}{\pi^*}\frac{\partial \pi^*}{\partial T_C^*}.
\end{aligned}
\tag{181}
$$

Based on equation (161) and on the values mentioned in tables 1, 2 and 3, equation (181) is negative. If the worker's labour tax rate is decreased by one percentage point, leading to an increased consumption tax rate, the entrepreneur's utility increases.

The effect on the workers' consumption is:

$$
\begin{aligned}
&\frac{\partial C^*}{\partial T_{LW}^*}\Big|_{dT_\pi^*=dG^*=0} = \frac{\partial C^*}{\partial T_{LW}^*} + \frac{\partial T_C^*}{\partial T_{LW}^*}\frac{\partial C^*}{\partial T_C^*} \\
&= \frac{\partial C^*}{\partial T_{LW}^*} - \frac{w^*\big(L^*+(N-L^*)\alpha\big)}{-T_\pi^* P^*\delta K^* + P^*(C^*+\delta K^*)}\frac{\partial C^*}{\partial T_C^*}.
\end{aligned}
\tag{182}
$$

Based on equation (160) and on the values mentioned in tables 1, 2 and 3, equation (182) is negative. If the worker's labour tax rate is decreased by one percentage point, leading to an increased consumption tax rate, the workers' consumption increases with 0.050 per cent.

The effect on the entrepreneurs' profit is:

$$\begin{aligned} &\frac{\partial \pi^*}{\partial T^*_{LW}}\Big|_{dT^*_\pi = dG^* = 0} = \frac{\partial \pi^*}{\partial T^*_{LW}} + \frac{\partial T^*_C}{\partial T^*_{LW}} \frac{\partial \pi^*}{\partial T^*_C} \\ &= \frac{\partial \pi^*}{\partial T^*_{LW}} - \frac{w^*\big(L^* + (N - L^*)\alpha\big)}{-T^*_\pi P^* \delta K^* + P^*(C^* + \delta K^*)} \frac{\partial \pi^*}{\partial T^*_C}. \end{aligned} \tag{183}$$

Based on equation (161) and on the values mentioned in tables 1, 2 and 3, equation (183) is negative. If the worker's labour tax rate is decreased by one percentage point, leading to an increased consumption tax rate, the entrepreneurs' profit increases with 0.327 per cent.

The effect on the wage rate is:

$$\begin{aligned} &\frac{\partial w^*}{\partial T^*_{LW}}\Big|_{dT^*_\pi = dG^* = 0} = \frac{\partial w^*}{\partial T^*_{LW}} + \frac{\partial T^*_C}{\partial T^*_{LW}} \frac{\partial w^*}{\partial T^*_C} \\ &= \frac{\partial w^*}{\partial T^*_{LW}} - \frac{w^*\big(L^* + (N - L^*)\alpha\big)}{-T^*_\pi P^* \delta K^* + P^*(C^* + \delta K^*)} \frac{\partial w^*}{\partial T^*_C}. \end{aligned} \tag{184}$$

Based on equation (159) and on the values mentioned in tables 1, 2 and 3, equation (184) is positive. If the worker's labour tax rate is decreased by one percentage point, leading to an increased consumption tax rate, the wage rate decreases with 0.002 per cent.

The effect on the employment rate is:

$$\begin{aligned} &\frac{\partial E^*}{\partial T^*_{LW}}\Big|_{dT^*_\pi = dG^* = 0} = \frac{\partial E^*}{\partial T^*_{LW}} + \frac{\partial T^*_C}{\partial T^*_{LW}} \frac{\partial E^*}{\partial T^*_C} \\ &= \frac{\partial E^*}{\partial T^*_{LW}} - \frac{w^*\big(L^* + (N - L^*)\alpha\big)}{-T^*_\pi P^* \delta K^* + P^*(C^* + \delta K^*)} \frac{\partial E^*}{\partial T^*_C}. \end{aligned} \tag{185}$$

Based on equation (158) and on the values mentioned in tables 1, 2 and 3, equation (185) is negative. If the worker's labour tax rate is decreased by one percentage point, leading to an increased consumption tax rate, the employment rate increases with 0.050 per cent.

The effect on the capital stock is:

$$\begin{aligned}\frac{\partial K^*}{\partial T^*_{LW}}\Big|_{dT^*_\pi = dG^* = 0} &= \frac{\partial K^*}{\partial T^*_{LW}} + \frac{\partial T^*_C}{\partial T^*_{LW}}\frac{\partial K^*}{\partial T^*_C} \\ &= \frac{\partial K^*}{\partial T^*_{LW}} - \frac{w^*\big(L^* + (N - L^*)\alpha\big)}{-T^*_\pi P^* \delta K^* + P^*(C^* + \delta K^*)}\frac{\partial K^*}{\partial T^*_C}.\end{aligned} \tag{186}$$

Based on equation (157) and on the values mentioned in tables 1, 2 and 3, equation (186) is positive. If the worker's labour tax rate is decreased by one percentage point, leading to an increased consumption tax rate, the capital stock decreases with 0.525 per cent.

The effect on the ratio between the entrepreneurs' profits and the value of the production is:

$$\begin{aligned}\frac{\partial \Omega^*_\pi}{\partial T^*_{LW}}\Big|_{dT^*_\pi = dG^* = 0} &= \frac{\partial \Omega^*_\pi}{\partial T^*_{LW}} + \frac{\partial T^*_C}{\partial T^*_{LW}}\frac{\partial \Omega^*_\pi}{\partial T^*_C} \\ &= \frac{\partial \Omega^*_\pi}{\partial T^*_{LW}} - \frac{w^*\big(L^* + (N - L^*)\alpha\big)}{-T^*_\pi P^* \delta K^* + P^*(C^* + \delta K^*)}\frac{\partial \Omega^*_\pi}{\partial T^*_C}.\end{aligned} \tag{187}$$

Based on equations (161), (172), (173), (175) and on the values mentioned in tables 1, 2 and 3, equation (187) is negative. If the worker's labour tax rate is decreased by one percentage point, leading to an increased consumption tax rate, the ratio between the entrepreneurs' profits and the value of the production increases with 0.278 per cent.

The effect on the ratio between the workers' labour income and the value of the production is:

$$\begin{aligned}\frac{\partial \Omega_{Lw}^*}{\partial T_{LW}^*}\Big|_{dT_\pi^*=dG^*=0} &= \frac{\partial \Omega_{Lw}^*}{\partial T_{LW}^*} + \frac{\partial T_C^*}{\partial T_{LW}^*}\frac{\partial \Omega_{Lw}^*}{\partial T_C^*} \\ &= \frac{\partial \Omega_{Lw}^*}{\partial T_{LW}^*} - \frac{w^*\big(L^* + (N-L^*)\alpha\big)}{-T_\pi^* P^* \delta K^* + P^*(C^* + \delta K^*)}\frac{\partial \Omega_{Lw}^*}{\partial T_C^*}.\end{aligned} \tag{188}$$

Based on equations (158), (159), (172), (173), (177) and on the values mentioned in tables 1, 2 and 3, equation (188) is positive. If the worker's labour tax rate is decreased by one percentage point, leading to an increased consumption tax rate, the ratio between the workers' labour income and the value of the production decreases with 0.051 per cent.

The effect on the gross income of the entrepreneur is:

$$\begin{aligned}\frac{\partial Y^*}{\partial T_{LW}^*}\Big|_{dT_\pi^*=dG^*=0} &= \frac{\partial Y^*}{\partial T_{LW}^*} + \frac{\partial T_C^*}{\partial T_{LW}^*}\frac{\partial Y^*}{\partial T_C^*} \\ &= \frac{\partial Y^*}{\partial T_{LW}^*} - \frac{w^*\big(L^* + (N-L^*)\alpha\big)}{-T_\pi^* P^* \delta K^* + P^*(C^* + \delta K^*)}\frac{\partial Y^*}{\partial T_C^*}.\end{aligned} \tag{189}$$

Based on equations (172), (173), (179) and on the values mentioned in tables 1, 2 and 3, equation (189) is negative. If the worker's labour tax rate is decreased by one percentage point, leading to an increased consumption tax rate, the gross income of the entrepreneur increases with 0.049 per cent.

Based on the above results, proposition 1 in chapter 5.2.1 holds.

D.3.2 Tax reform: Shifting the taxation from profit to consumption

Assume that the labour taxation and the public expenditures are kept constant. Thus when the profit taxation changes, the tax rate on consumption must change in order to keep the government's budget balanced.

Based on equations (164) and (165), the effect of this reform proposal on the utility of a worker can be estimated. The change in the profit taxation has a direct effect on the utility as well as an indirect effect, generated by the change in consumption taxation. The total effect on the utility is

$$\begin{aligned}\frac{\partial U_W^*}{\partial T_\pi^*}\Big|_{dT_{LW}^*=dG^*=0} &= \frac{\partial U_W^*}{\partial T_\pi^*} + \frac{\partial T_C^*}{\partial T_\pi^*}\frac{\partial U_W^*}{\partial T_C^*} \\ &= \frac{1}{C^*}\frac{\partial C^*}{\partial T_\pi^*} - \frac{\psi}{N-L^*}\frac{\partial L^*}{\partial T_\pi^*} \\ &- \frac{P^*F(K^*,L^*) - w^*L^* - (1+T_C^*)P^*\delta K^*}{-T_\pi^* P^*\delta K^* + P^*(C^*+\delta K^*)} \\ &\times \left(\frac{1}{C^*}\frac{\partial C^*}{\partial T_C^*} - \frac{\psi}{N-L^*}\frac{\partial L^*}{\partial T_C^*}\right).\end{aligned} \tag{190}$$

Based on equations (158), (160) and on the values mentioned in tables 1, 2 and 3, equation (190) is positive. If the profit tax rate is decreased by one percentage point, leading to an increased consumption tax rate, the worker's utility decreases.

Correspondingly, regarding the entrepreneurs, the total effect on the utility, based on equations (164) and (167), is

$$\begin{aligned}\frac{\partial U_E^*}{\partial T_\pi^*}\Big|_{dT_{LW}^*=dG^*=0} &= \frac{\partial U_E^*}{\partial T_\pi^*} + \frac{\partial T_C^*}{\partial T_\pi^*}\frac{\partial U_E^*}{\partial T_C^*} \\ &= \frac{1}{\pi^*}\frac{\partial \pi^*}{\partial T_\pi^*} - \frac{P^*F(K^*,L^*) - w^*L^* - (1+T_C^*)P^*\delta K^*}{-T_\pi^* P^*\delta K^* + P^*(C^*+\delta K^*)} \times \frac{1}{\pi^*}\frac{\partial \pi^*}{\partial T_C^*}.\end{aligned} \tag{191}$$

Based on equation (161) and on the values mentioned in tables 1, 2 and 3, equation (191) is negative. If the profit tax rate is decreased by one percentage point, leading to an increased consumption tax rate, the entrepreneur's utility increases.

The effect on the workers' consumption is:

$$\begin{aligned}
&\frac{\partial C^*}{\partial T_\pi^*}|_{dT_{LW}^*=dG^*=0} = \frac{\partial C^*}{\partial T_\pi^*} + \frac{\partial T_C^*}{\partial T_\pi^*}\frac{\partial C^*}{\partial T_C^*} \\
&= \frac{\partial C^*}{\partial T_\pi^*} - \frac{P^*F(K^*,L^*) - w^*L^* - (1+T_C^*)P^*\delta K^*}{-T_\pi^*P^*\delta K^* + P^*(C^*+\delta K^*)} \times \frac{\partial C^*}{\partial T_C^*}.
\end{aligned} \tag{192}$$

Based on equation (160) and on the values mentioned in tables 1, 2 and 3, equation (192) is positive. If the profit tax rate is decreased by one percentage point, leading to an increased consumption tax rate, the workers' consumption decreases with 0.001 per cent.

The effect on the entrepreneurs' profit is:

$$\begin{aligned}
&\frac{\partial \pi^*}{\partial T_\pi^*}|_{dT_{LW}^*=dG^*=0} = \frac{\partial \pi^*}{\partial T_\pi^*} + \frac{\partial T_C^*}{\partial T_\pi^*}\frac{\partial \pi^*}{\partial T_C^*} \\
&= \frac{\partial \pi^*}{\partial T_\pi^*} - \frac{P^*F(K^*,L^*) - w^*L^* - (1+T_C^*)P^*\delta K^*}{-T_\pi^*P^*\delta K^* + P^*(C^*+\delta K^*)} \times \frac{\partial \pi^*}{\partial T_C^*}.
\end{aligned} \tag{193}$$

Based on equation (161) and on the values mentioned in tables 1, 2 and 3, equation (193) is negative. If the profit tax rate is decreased by one percentage point, leading to an increased consumption tax rate, the entrepreneurs' profit increases with 1.250 per cent.

The effect on the wage rate is:

$$\begin{aligned}
&\frac{\partial w^*}{\partial T_\pi^*}|_{dT_{LW}^*=dG^*=0} = \frac{\partial w^*}{\partial T_\pi^*} + \frac{\partial T_C^*}{\partial T_\pi^*}\frac{\partial w^*}{\partial T_C^*} \\
&= \frac{\partial w^*}{\partial T_\pi^*} - \frac{P^*F(K^*,L^*) - w^*L^* - (1+T_C^*)P^*\delta K^*}{-T_\pi^*P^*\delta K^* + P^*(C^*+\delta K^*)} \times \frac{\partial w^*}{\partial T_C^*}.
\end{aligned} \tag{194}$$

Based on equation (159) and on the values mentioned in tables 1, 2 and 3, equation (194) is positive. If the profit tax rate is decreased by one percentage point, leading to an increased consumption tax rate, the wage rate decreases with 3.266 $\times 10^{-6}$ per cent.

The effect on the employment rate is:

$$\begin{aligned}\frac{\partial E^*}{\partial T_\pi^*}|_{dT_{LW}^*=dG^*=0} &= \frac{\partial E^*}{\partial T_\pi^*} + \frac{\partial T_C^*}{\partial T_\pi^*}\frac{\partial E^*}{\partial T_C^*} \\ &= \frac{\partial E^*}{\partial T_\pi^*} - \frac{P^*F(K^*,L^*) - w^*L^* - (1+T_C^*)P^*\delta K^*}{-T_\pi^* P^*\delta K^* + P^*(C^*+\delta K^*)} \times \frac{\partial E^*}{\partial T_C^*}.\end{aligned} \tag{195}$$

Based on equation (158) and on the values mentioned in tables 1, 2 and 3, equation (195) is negative. If the profit tax rate is decreased by one percentage point, leading to an increased consumption tax rate, the employment rate increases with 2.560 $\times 10^{-5}$ per cent.

The effect on the capital stock is:

$$\begin{aligned}\frac{\partial K^*}{\partial T_\pi^*}|_{dT_{LW}^*=dG^*=0} &= \frac{\partial K^*}{\partial T_\pi^*} + \frac{\partial T_C^*}{\partial T_\pi^*}\frac{\partial K^*}{\partial T_C^*} \\ &= \frac{\partial K^*}{\partial T_\pi^*} - \frac{P^*F(K^*,L^*) - w^*L^* - (1+T_C^*)P^*\delta K^*}{-T_\pi^* P^*\delta K^* + P^*(C^*+\delta K^*)} \times \frac{\partial K^*}{\partial T_C^*}.\end{aligned} \tag{196}$$

Based on equation (157) and on the values mentioned in tables 1, 2 and 3, equation (196) is positive. If the profit tax rate is decreased by one percentage point, leading to an increased consumption tax rate, the capital stock decreases with 0.001 per cent.

The effect on the ratio between the entrepreneurs' profits and the value of the production is:

$$\begin{aligned}\frac{\partial \Omega_\pi^*}{\partial T_\pi^*}|_{dT_{LW}^*=dG^*=0} &= \frac{\partial \Omega_\pi^*}{\partial T_\pi^*} + \frac{\partial T_C^*}{\partial T_\pi^*}\frac{\partial \Omega_\pi^*}{\partial T_C^*} \\ &= \frac{\partial \Omega_\pi^*}{\partial T_\pi^*} - \frac{P^*F(K^*,L^*) - w^*L^* - (1+T_C^*)P^*\delta K^*}{-T_\pi^* P^*\delta K^* + P^*(C^*+\delta K^*)} \times \frac{\partial \Omega_\pi^*}{\partial T_C^*}.\end{aligned} \tag{197}$$

Based on equations (161), (172), (173), (175) and on the values mentioned in tables 1, 2 and 3, equation (197) is negative. If the profit tax rate is decreased by one percentage point, leading to an increased consumption tax rate, the ratio between the entrepreneurs' profits and the value of the production increases with 1.250 per cent.

The effect on the ratio between the workers' labour income and the value of the production is:

$$\begin{aligned}
&\frac{\partial \Omega^*_{Lw}}{\partial T^*_\pi}\Big|_{dT^*_{LW}=dG^*=0} = \frac{\partial \Omega^*_{Lw}}{\partial T^*_\pi} + \frac{\partial T^*_C}{\partial T^*_\pi}\frac{\partial \Omega^*_{Lw}}{\partial T^*_C} \\
&= \frac{\partial \Omega^*_{Lw}}{\partial T^*_\pi} - \frac{P^*F(K^*,L^*) - w^*L^* - (1+T^*_C)P^*\delta K^*}{-T^*_\pi P^*\delta K^* + P^*(C^*+\delta K^*)} \times \frac{\partial \Omega^*_{Lw}}{\partial T^*_C}.
\end{aligned} \tag{198}$$

Based on equations (158), (159), (172), (173), (177) and on the values mentioned in tables 1, 2 and 3, equation (198) is positive. If the profit tax rate is decreased by one percentage point, leading to an increased consumption tax rate, the ratio between the workers' labour income and the value of the production decreases with 2.660 $\times 10^{-5}$ per cent.

The effect on the gross income of the entrepreneur is:

$$\begin{aligned}
&\frac{\partial Y^*}{\partial T^*_\pi}\Big|_{dT^*_{LW}=dG^*=0} = \frac{\partial Y^*}{\partial T^*_\pi} + \frac{\partial T^*_C}{\partial T^*_\pi}\frac{\partial Y^*}{\partial T^*_C} \\
&= \frac{\partial Y^*}{\partial T^*_\pi} - \frac{P^*F(K^*,L^*) - w^*L^* - (1+T^*_C)P^*\delta K^*}{-T^*_\pi P^*\delta K^* + P^*(C^*+\delta K^*)} \times \frac{\partial Y^*}{\partial T^*_C}.
\end{aligned} \tag{199}$$

Based on equations (172), (173), (179) and on the values mentioned in tables 1, 2 and 3, equation (199) is negative. If the profit tax rate is decreased by one percentage point, leading to an increased consumption tax rate, the gross income of the entrepreneur increases with 2.340 $\times 10^{-5}$ per cent.

Based on the above results, proposition 2 in chapter 5.2.2 holds.

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